All-Pairs Approximate Shortest Paths and Distance Oracle Preprocessing

Christian Sommer
csommer@apple.com
http://www.chsommer.com

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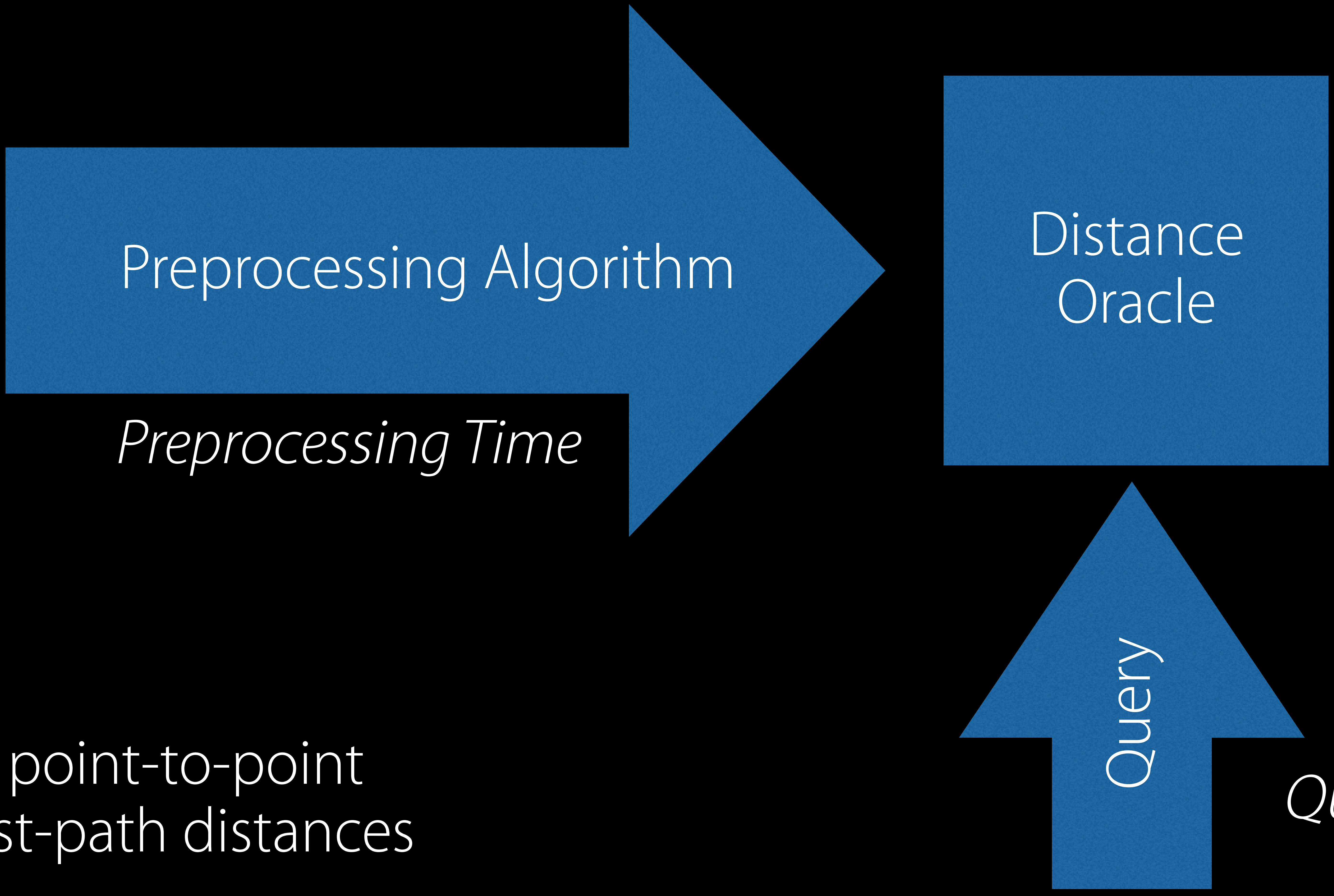
Distance Oracle

Thorup & Zwick (STOC'01)

Graph $G=(V, E)$

$n := |V|$ $m := |E|$



Preprocessing Time

Space

Query Time

$O(1)$

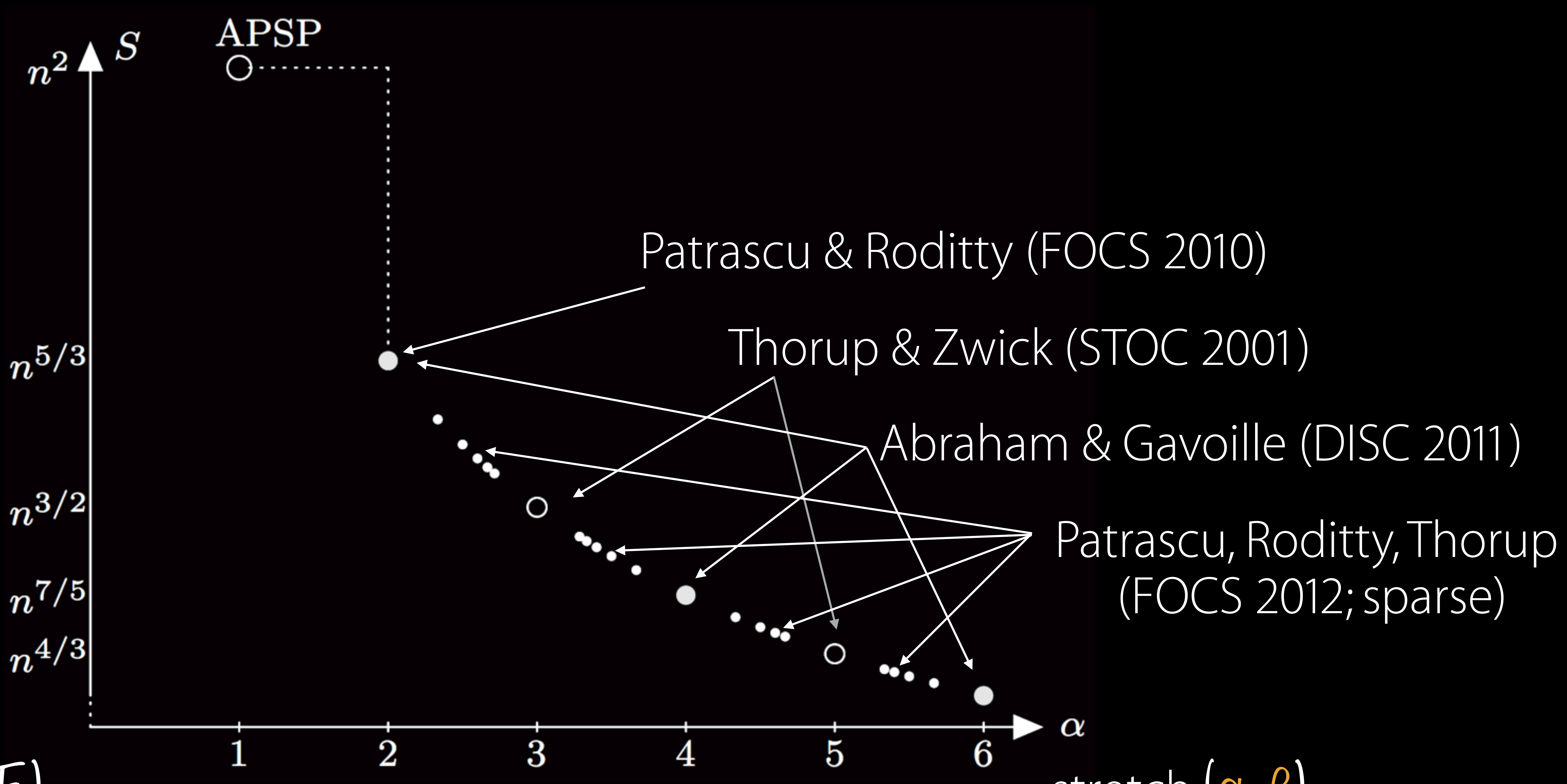
Data structure for point-to-point *approximate* shortest-path distances

stretch (α, β)

$$d(s,t) \leq X \leq \alpha \cdot d(s,t) + \beta$$

Space vs. Stretch

space



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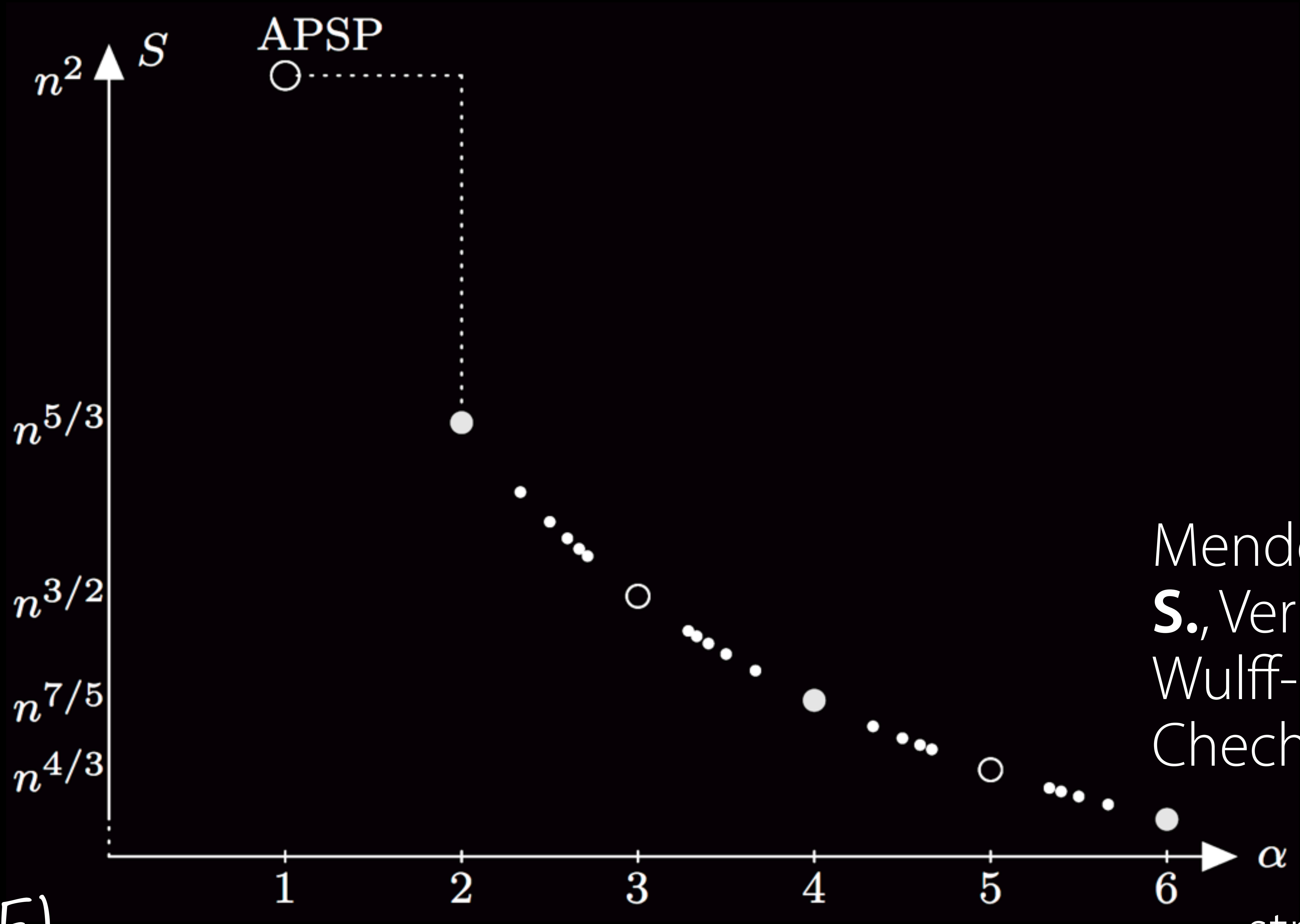
Space vs. Stretch vs. Query Time

space



Mendel & Naor (FOCS 2006)
S., Verbin, Yu (FOCS 2009)
Wulff-Nilsen (SODA 2013)
Chechik (STOC 2014, 2015)

Graph $G=(V, E)$

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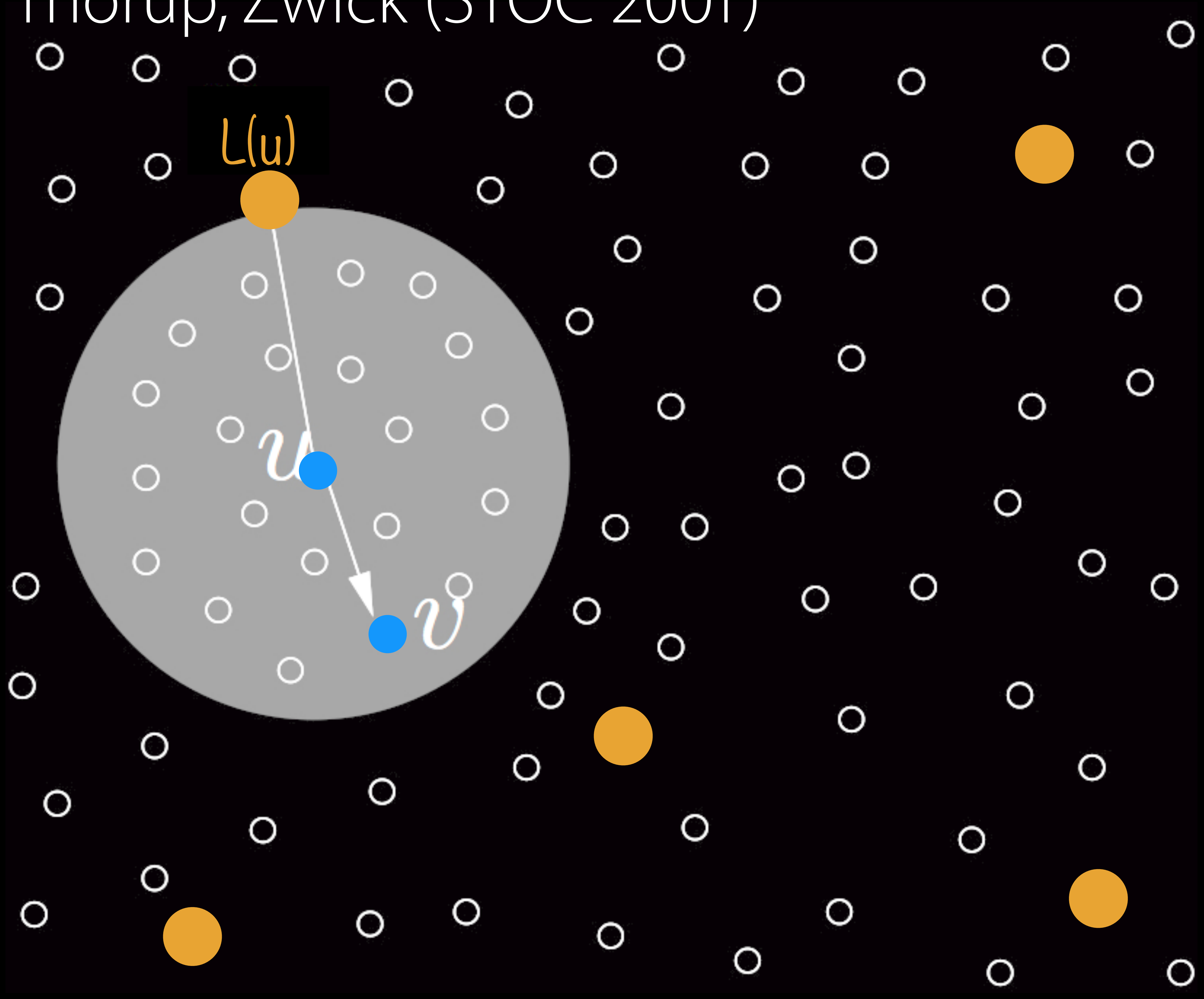
APASP/Preprocessing (Dense Graphs) unweighted

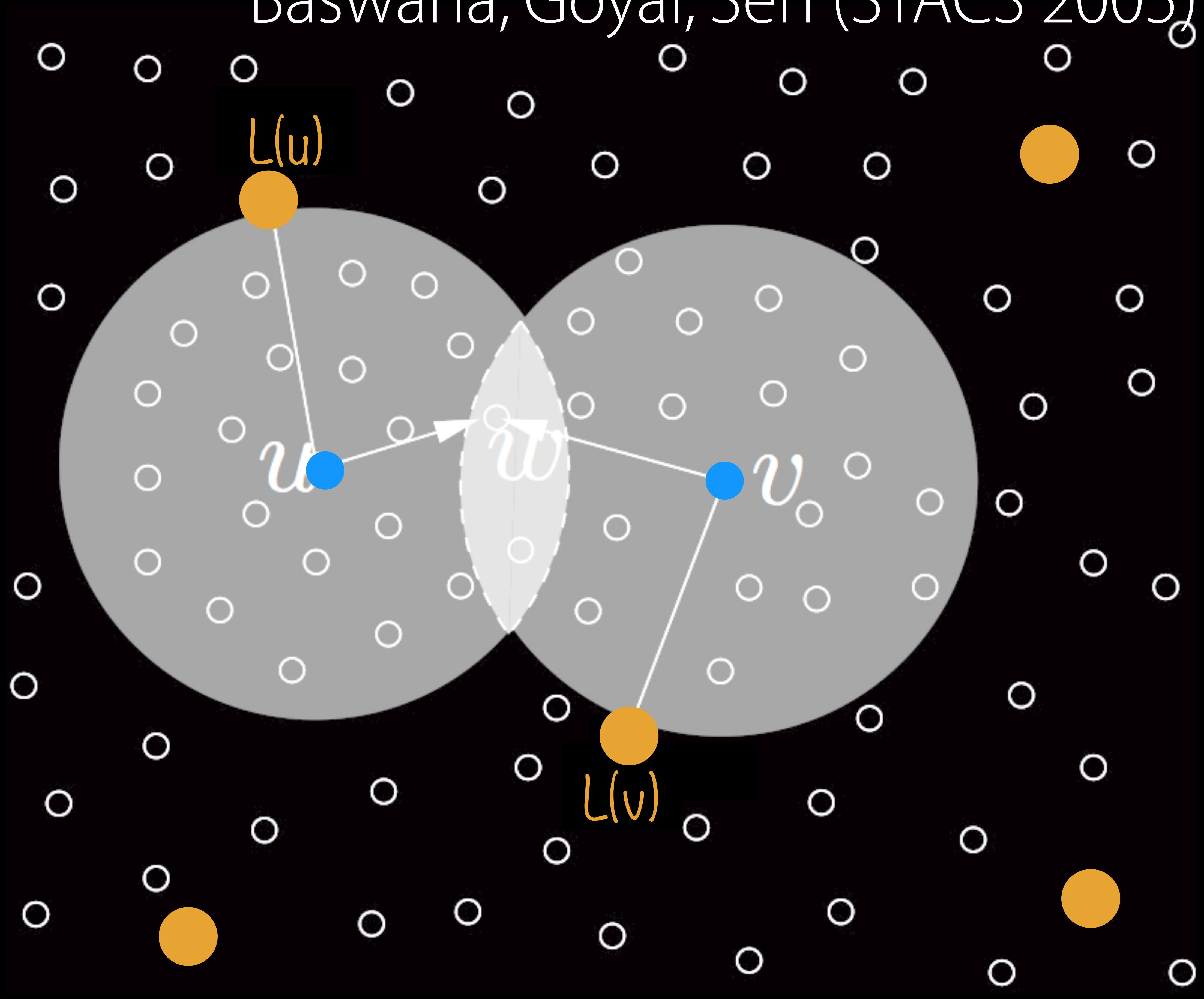
Stretch	Time $\tilde{O}(\cdot)$	Space $\tilde{O}(\cdot)$	
(1,2)	$n^{5/2}$	n^2	Aingworth, Chekuri, Indyk, Motwani (SODA 1996)
(1,2)	$n^{7/3}$	n^2	Dor, Halperin, Zwick (FOCS 1996)
(3,0)	n^2	n^2	Cohen, Zwick (SODA 1997)
(3,0)	$n^{5/2}$	$n^{3/2}$	Thorup, Zwick (STOC 2001)
(3,0)	n^2	$n^{3/2}$	Baswana, Sen (SODA 2004); B., Kavitha (FOCS 2006)
(3,10)	$n^{23/12} + m$	$n^{3/2}$	Baswana, Gaur, Sen, Upadhyay (ICALP 2008)
(2,1)	n^2	n^2	Berman, Kasiviswanathan (WADS 2007)
(2,1)	$n^{8/3}$	$n^{5/3}$	Baswana, Goyal, Sen (STACS 2005) *(space bound implicit)*
(2,3)	n^2	$n^{5/3}$	
(2,1)	poly	$n^{5/3}$	Patrascu, Roditty (FOCS 2010)
(2,1)	n^2	$n^{5/3}$	NEW

Landmarks and Balls

Thorup, Zwick (STOC 2001)



Baswana, Goyal, Sen (STACS 2005)



landmarks (random sample, probability p, keep distances to all np landmarks)

balls (all nodes closer than nearest landmark, expected size $1/p$; ***use ball or triangulate***)

ball intersection (store all nodes with intersecting balls, expected size $1/p^2$)

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Dominating Sets

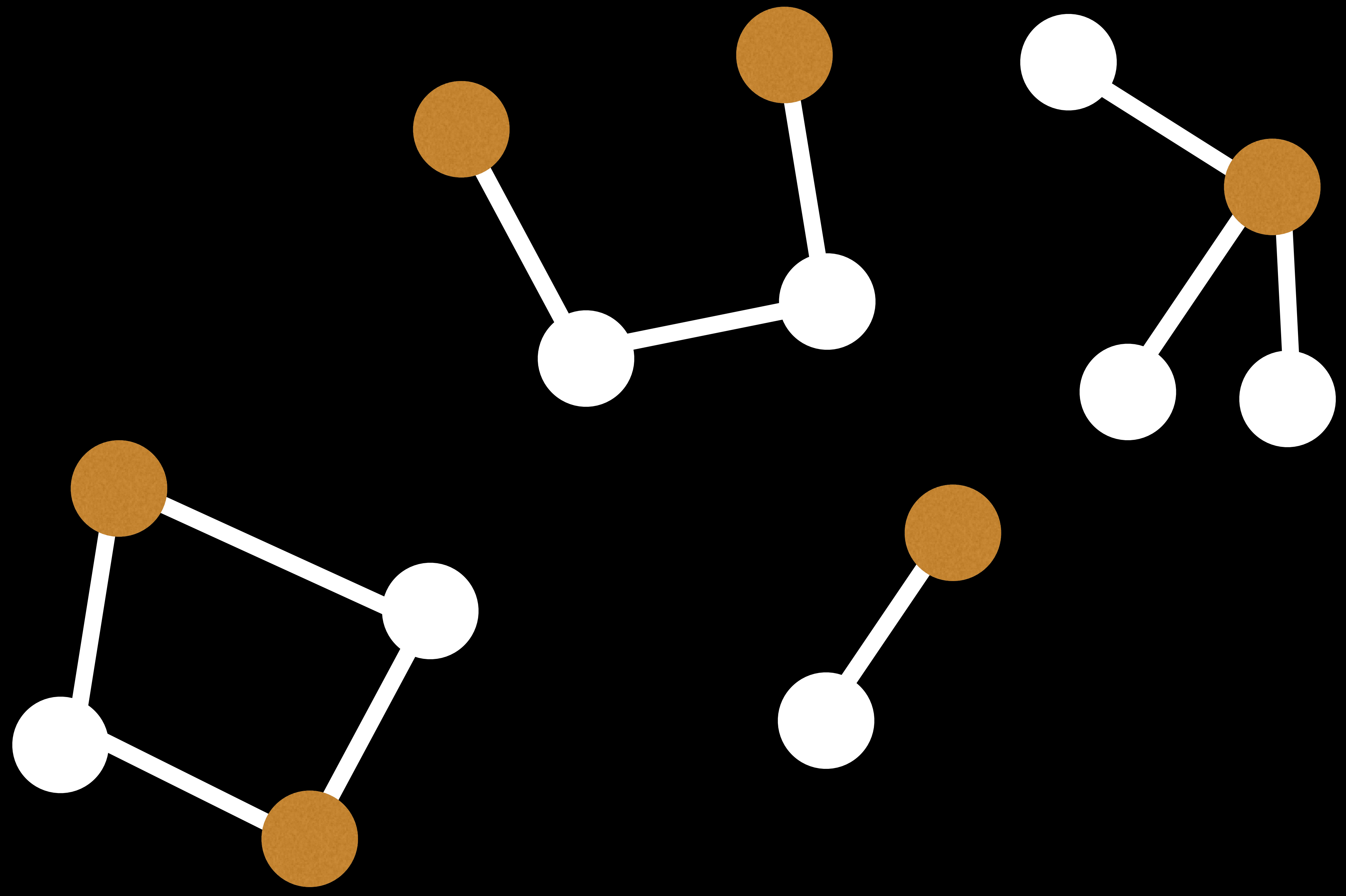
each node or a neighbor is in the dominating set

Aingworth, Chekuri, Indyk, Motwani
(SODA 1996)

Dominating Set for

High-Degree Nodes $deg(v) > \delta$

Size: $\sim n / \delta$



How to exploit Dominating Sets

Berman, Kasiviswanathan (WADS 2007)

for $\delta = n, n/2, n/4, \ldots, n/2^i, \ldots$

High-Degree Nodes $deg(v) > \delta$

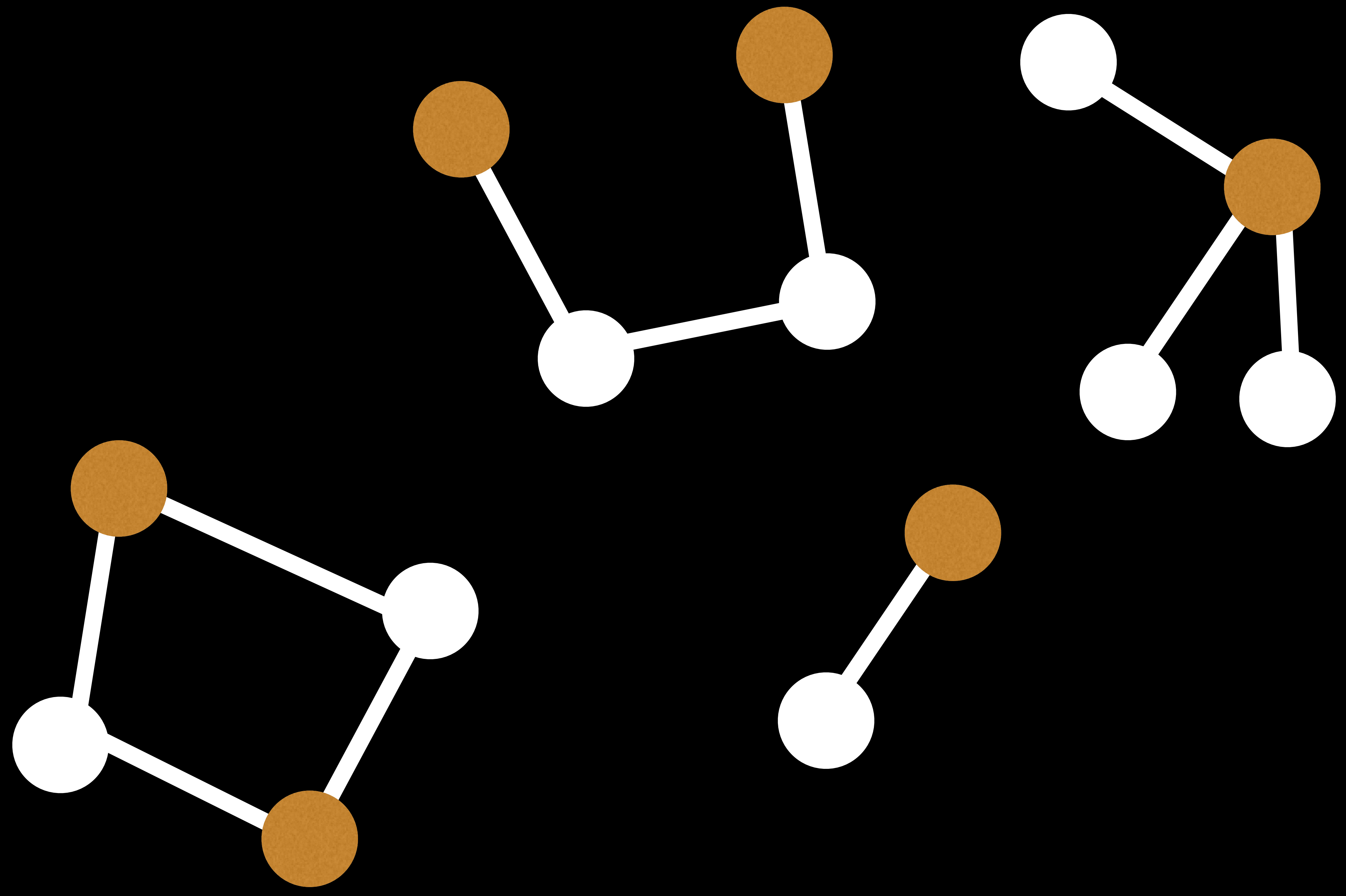
Dominating Set, size $\sim n / \delta$

BFS Tree from each ***dominator***

in low-degree graph $deg(v) < 2\delta$

store distances to all dominators

nearest dominator: ***landmark***



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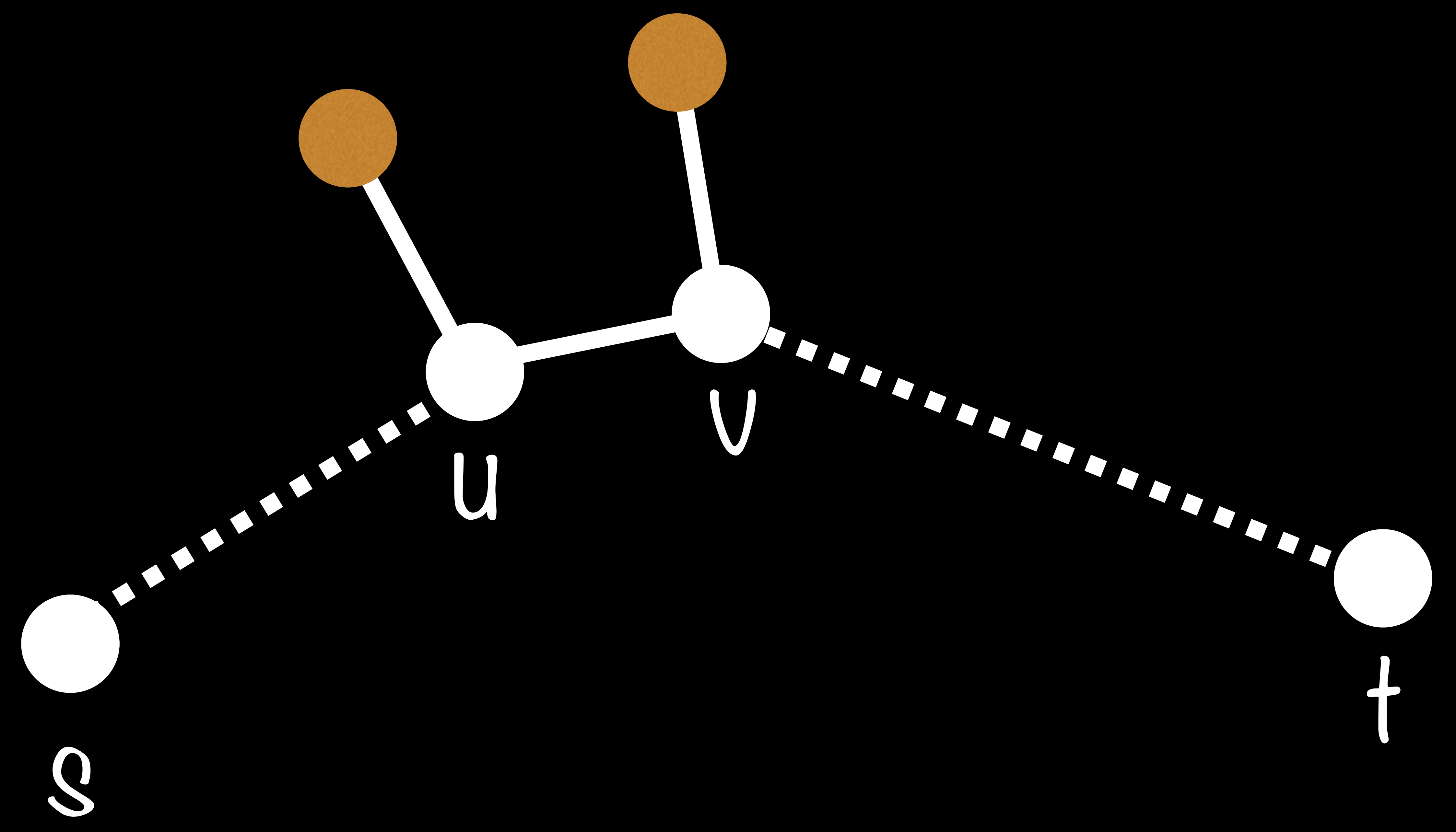

edge uv has highest degree

among edges on shortest s-t path

(will return min among all levels δ)

for $\delta = n, n/2, n/4, \ldots, n/2^i, \ldots$

High-Degree Nodes $deg(v) > \delta$

Dominating Set, size $\sim n / \delta$

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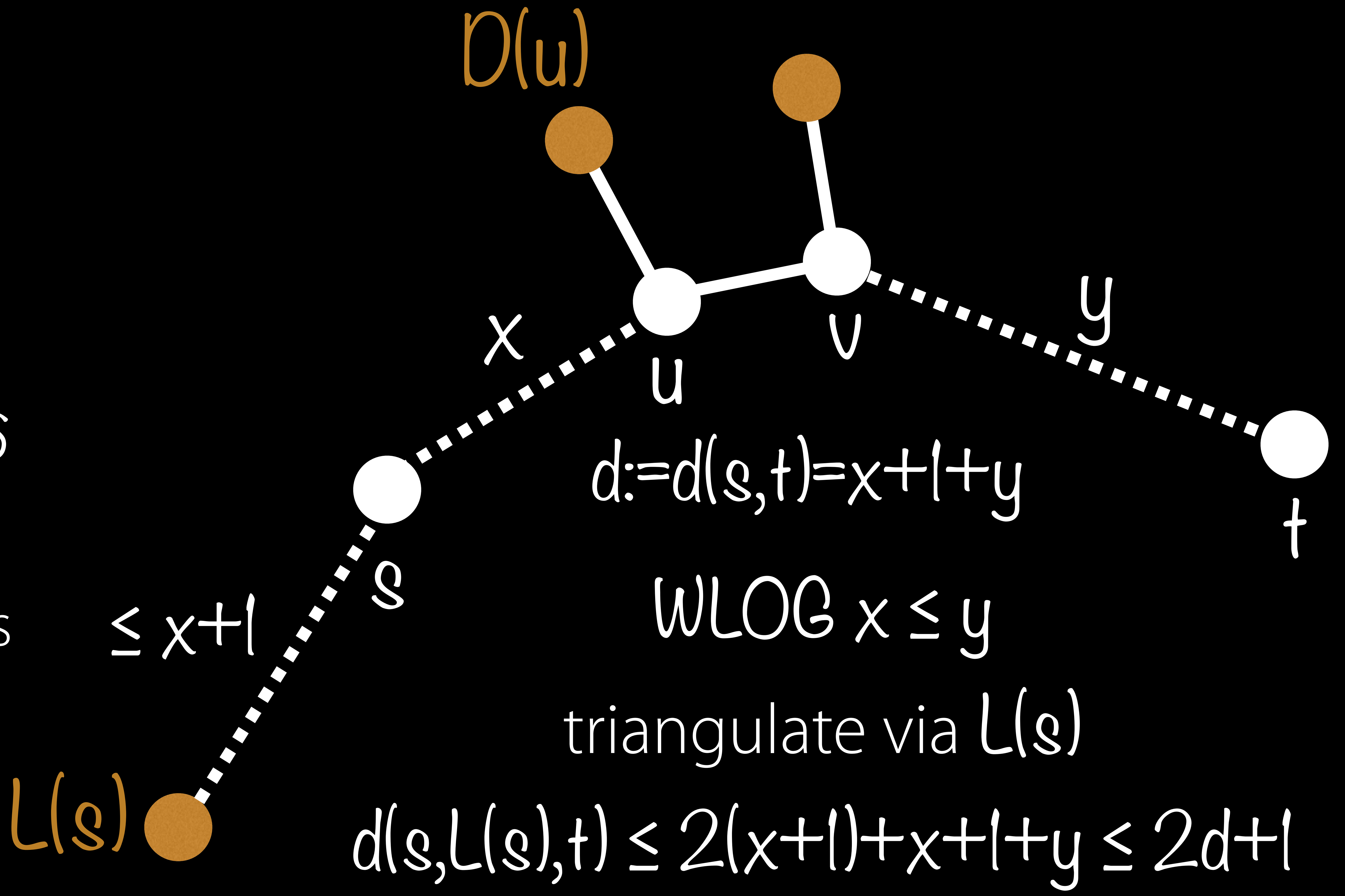

log n levels

for $\delta = n, n/2, n/4, \ldots, n/2^i, \ldots$

High-Degree Nodes $deg(v) > \delta$

Dominating Set, size $\sim n / \delta$

BFS Tree from each *dominator*

in low-degree graph $deg(v) < 2\delta$

store distances to all dominators

nearest dominator: *landmark*

can't afford to query all log n levels

but don't know $deg(uv)$

time $m + n\delta$

$n \cdot \delta$ edges, hence *time* n^2

n^2/δ *space*

stop at $\delta = n^{1/3}$

handle remaining

sparse graph separately

Baswana, Goyal, Sen, 2005

log n levels

for $\delta = n, n/2, n/4, \ldots, n/2^i, \ldots$

High-Degree Nodes $deg(v) > \delta$

Dominating Set, size $\sim n / \delta$

BFS Tree from each *dominator*

in low-degree graph $deg(v) < 2\delta$

store distances to all dominators

nearest dominator: *landmark*

can't afford to query all log n levels

but don't know $deg(uv)$

Tight. *(Abboud and Bodwin, STOC 2016)*

Spanner (Woodruff, ICALP 2010)

Always include $n^{4/3}$ edges of (6) spanner

Portal Selection

Landmark at level $n/2^i$ is a ***portal*** for s

if it is ***closer*** than all landmarks at levels $j < i$.

Keep nearest 3 portals per node.

Portal Selection

Landmark at level $n/2^i$ is a ***portal*** for s

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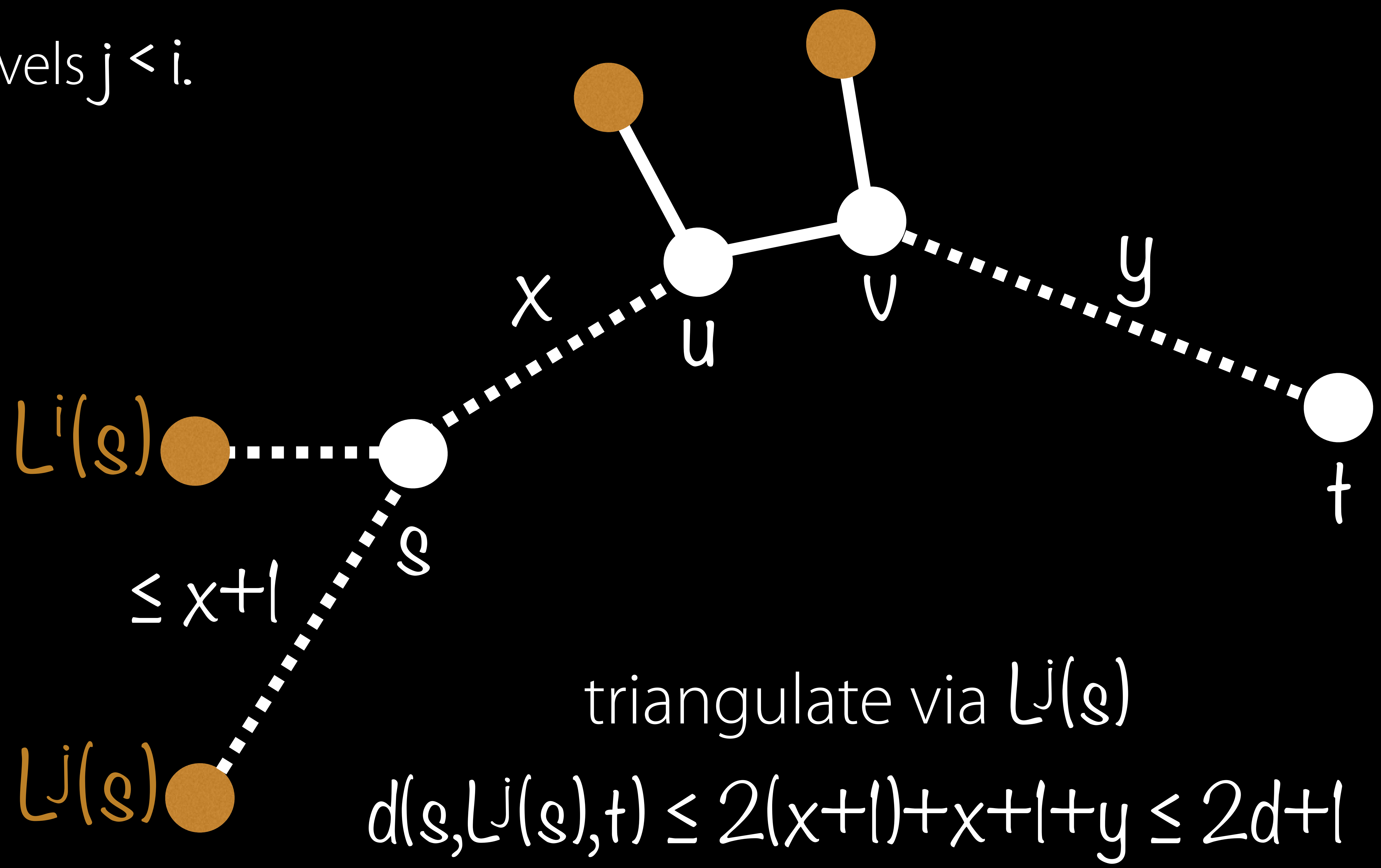
$i > j$ means that the BFS from $L^i(s)$

runs in a subgraph of

the BFS from $L^j(s)$

better to triangulate via $L^j(s)$

unless $d(s,L^i(s)) < d(s, L^j(s))$



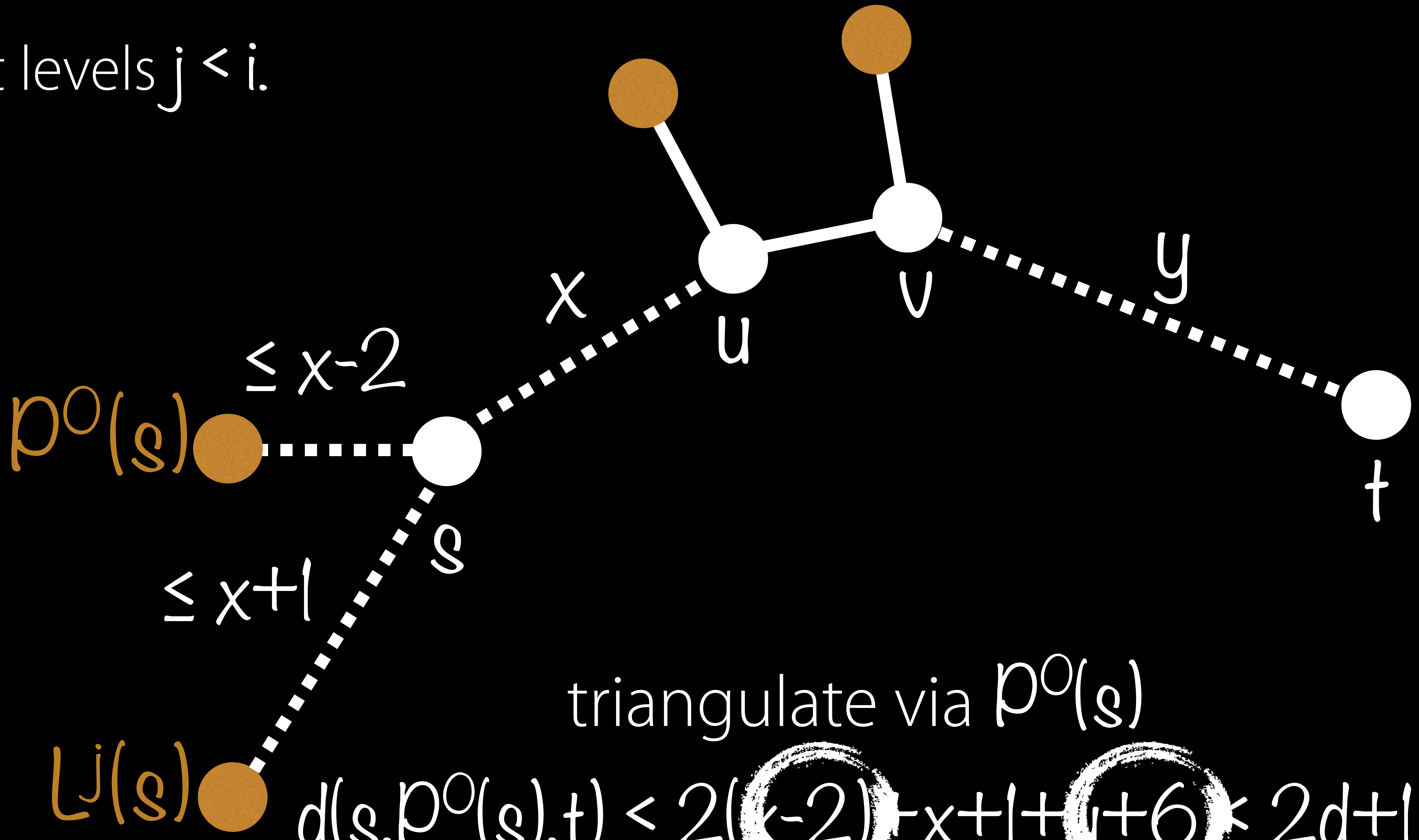
$\beta/2 = 3$ Portals

Landmark at level $n/2^i$ is a ***portal*** for s

if it is ***closer*** than all landmarks at levels $j < i$.

If the best $L^j(s)$ is
not among the portals,
it must be ***far*** away:

$d(s, L^j(s)) \geq d(s, P^O(s)) + 3$



triangulate via $P^O(s)$

$d(s, P^O(s), t) \leq 2(x-2)+x+1+y+6 \leq 2d+1$

Preprocessing

compute (1,6) spanner [Woodruff]

for $\delta = n, n/2, n/4, \ldots, n/2^i, \ldots n^{1/3}$

High-Degree Nodes $deg(v) > \delta$

Dominating Set of size n / δ

BFS Tree from each ***dominator***

in low-degree graph $deg(v) < 2\delta$
plus edges of spanner

store distances to all dominators
nearest dominator: *landmark*

for each node: nearest 3 ***portals***

compute oracle for $deg(v) < 2n^{1/3}$ graph
[Baswana, Goyal, Sen]

Query d(s,t)

return min among

6 triangulations via top 3 ***portals***
and estimate from sparse oracle

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Thanks!
Grazie!