

# On Balanced Separators in Road Networks

Aaron Schild

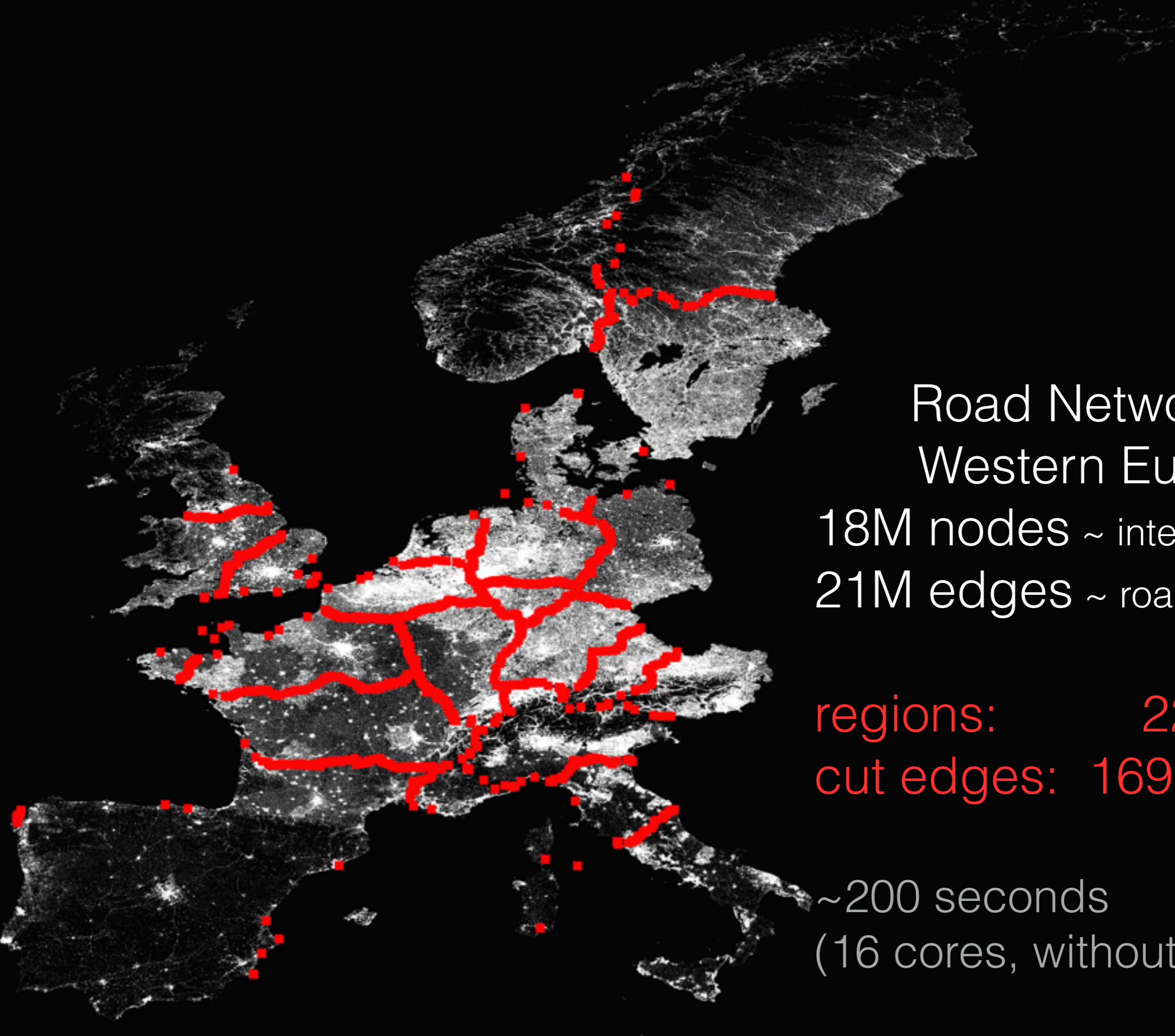
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# Road Network of Western Europe

18M nodes ~ intersections  
21M edges ~ road segments

regions: 22  
cut edges: 1694

~200 seconds  
(16 cores, without I/O)

# Applications

- Shortest-Path Queries, Route Planning
  - Customizable Route Planning [DGPW SEA'11]
  - Customizable Contraction Hierarchies [DSW SEA'14]
- Optimization Algorithms
  - *cf.* separators and  $r$ -divisions in planar graphs

# Related Work: Experimental

- Various Graph Partitioners and Heuristics
- Kernighan-Lin 1970, Fiduccia-Mattheyses 1982
- METIS [KK '95], SCOTCH, JOSTLE, CHACO,...
- PUNCH [DGRW '11]
- KaPPa, KaSPar, KaHiP, Buffoon [SS '11,'12,'13]

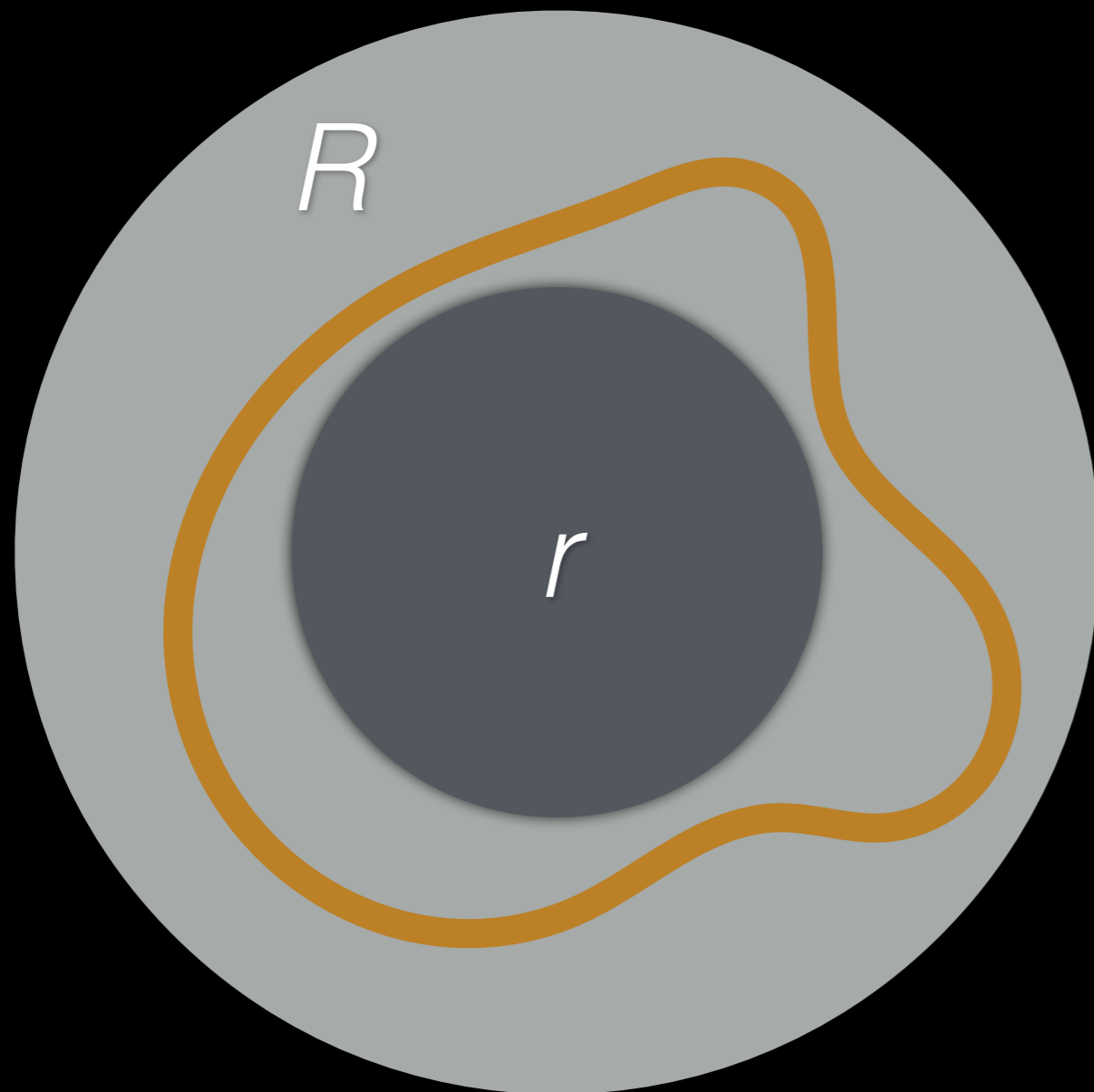
# PUNCH

[Delling, Goldberg, Razenshteyn, Werneck, 2011]

- road networks have *very* small separators
- planar: square root [LT79]    road networks: *cube* root
- *natural* cuts: rivers, mountains, freeways, etc.
- two-phase algorithm:
  - filtering: compute *natural* cuts (candidates)
  - assembly: choose among (combinations of) candidates

# PUNCH

[Delling, Goldberg, Razenshteyn, Werneck, 2011]



random center (node)

BFS balls

- source:  $<$  radius  $r$
- sink:  $>$  radius  $R$

max flow / min cut

# Related Work: Theory

- Separators and  $r$ -divisions in planar graphs
- Approximation Algorithms for Sparsest Cut:
- Spectral Cut: compute second-largest Eigenvector of Laplacian, embedding, sort nodes, sweep cut
- Refine embedding by maximum flow computations  
[KRV'06, AK'07, OSVV'08, S'09]

# Related Work

*Finding a sparse cut using max-flow is easy*

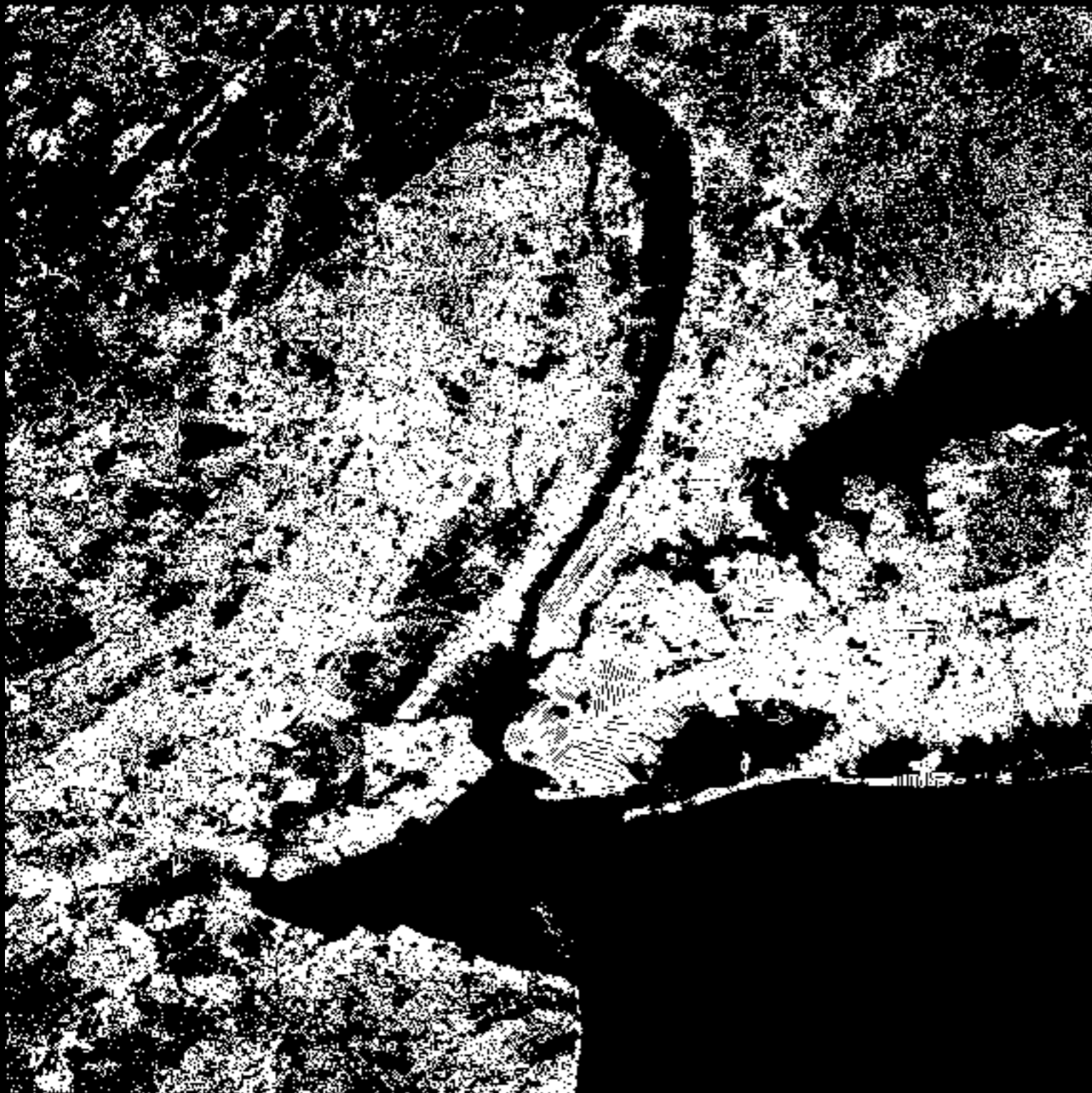
***if** you can identify many vertices on each side of a good cut.*

*The difficulty, of course, is in identifying these vertices.*

[Khandekar, Rao, Vazirani; STOC'06]



# Inertial Flow



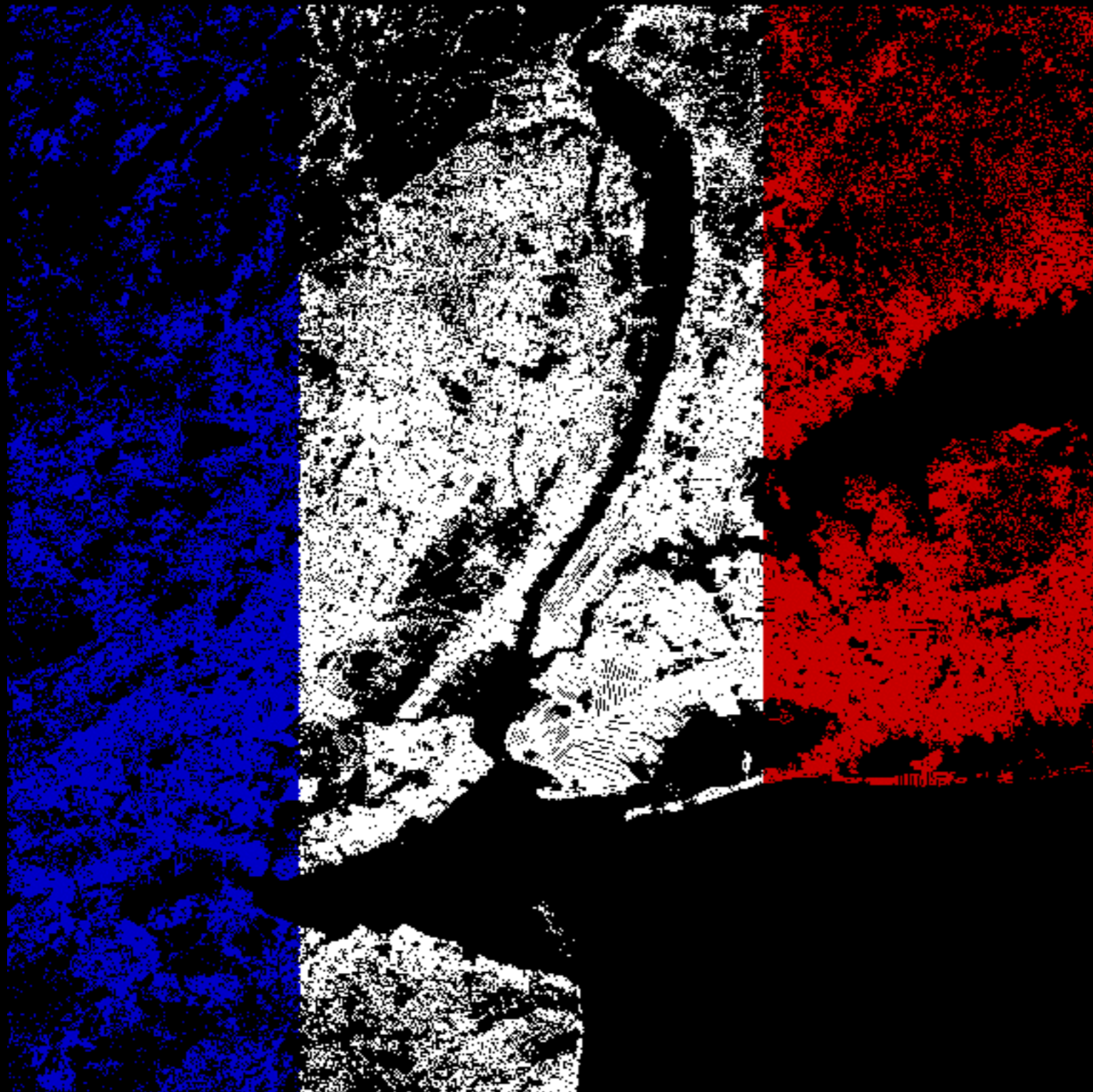
sort  $n$  nodes by latitude  
(or longitude,  
or linear combination)

balance (say  $b=1/4$ )  
 $bn$  sources  
 $bn$  sinks

max flow / min cut

recurse

# Inertial Flow



sort  $n$  nodes by latitude  
(or longitude,  
or linear combination)

balance (say  $b=1/4$ )

$bn$  sources

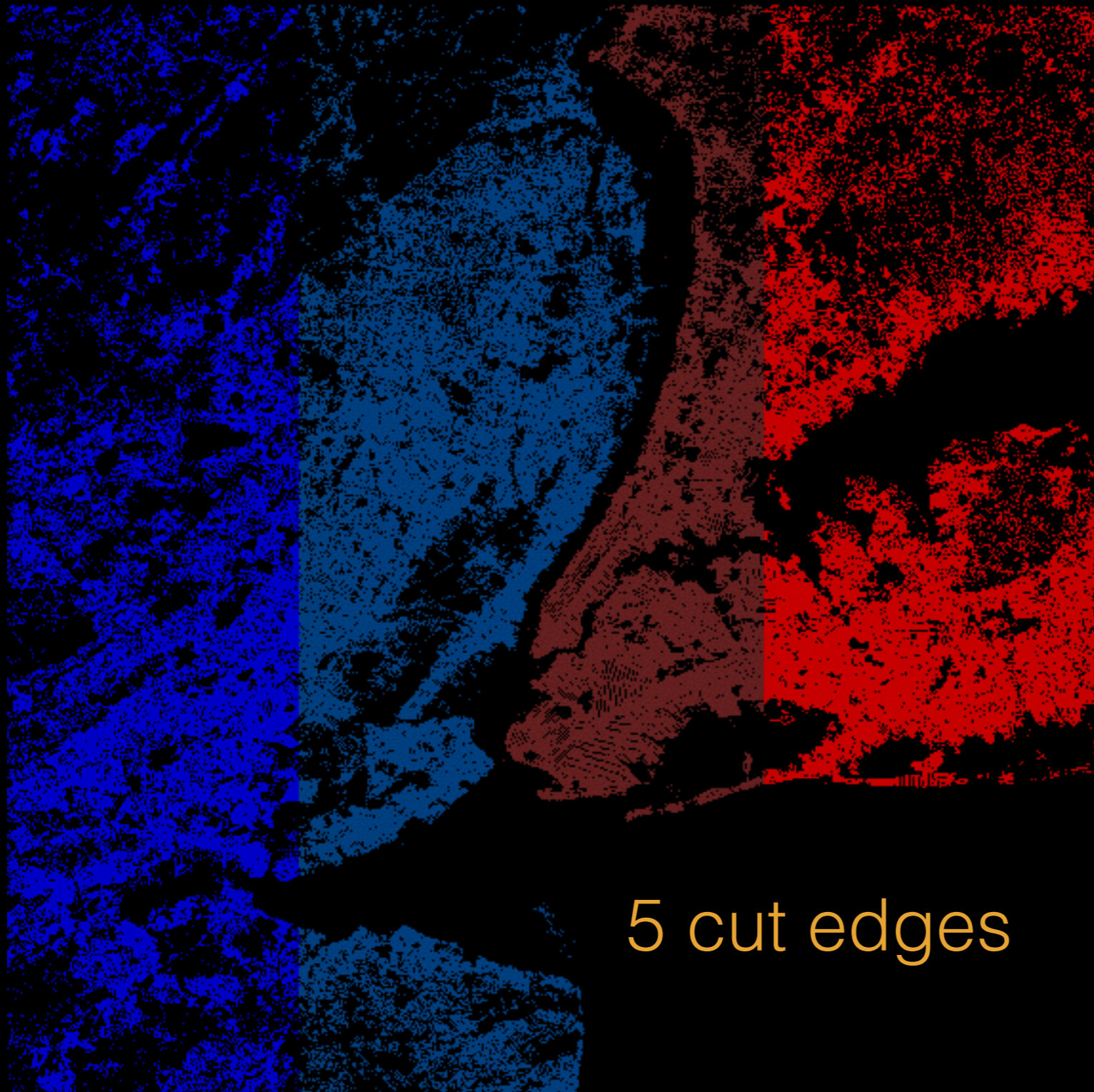
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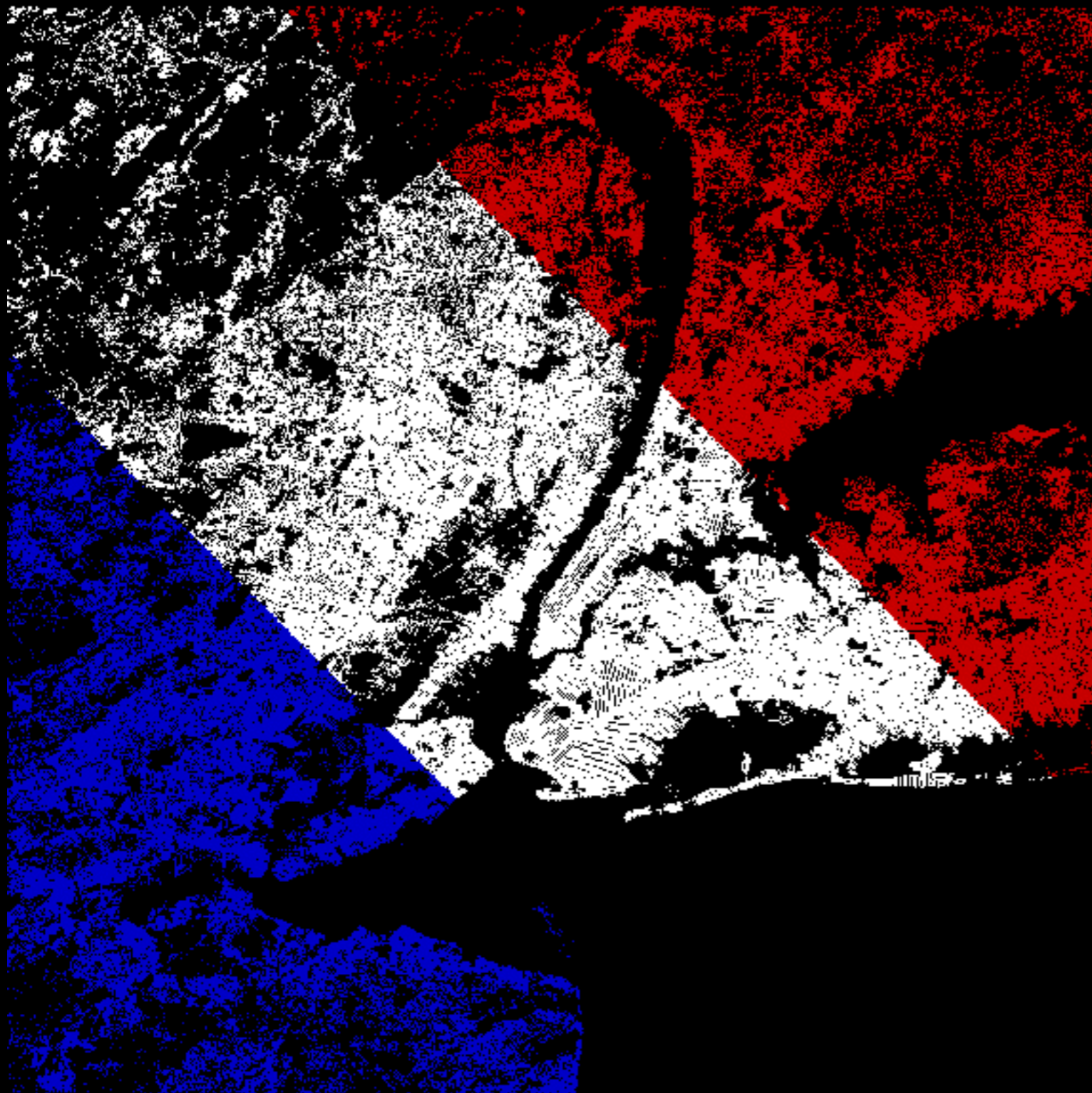
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# Inertial Flow



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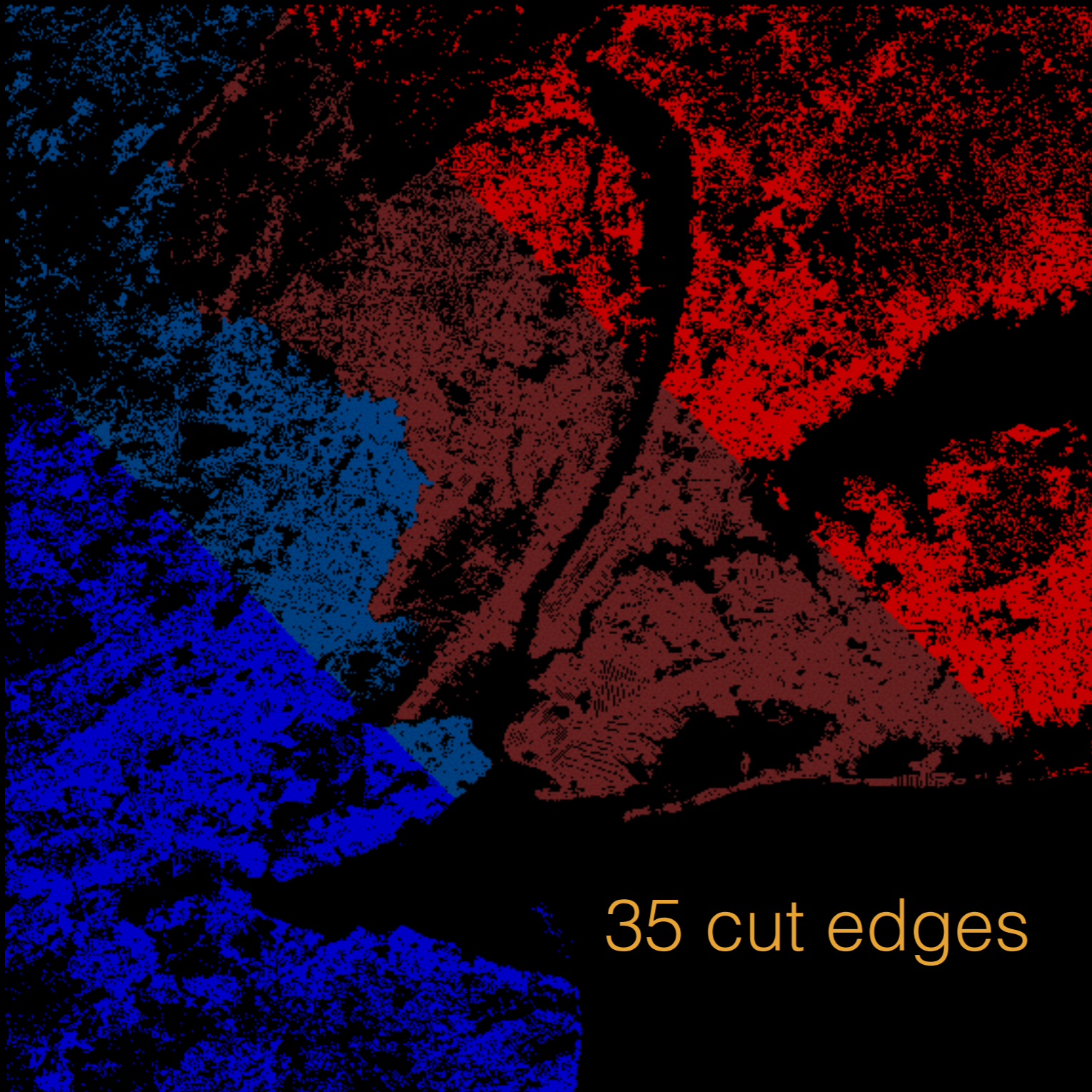
balance (say  $b=1/4$ )  
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max flow / min cut

recurse



# Inertial Flow



35 cut edges

sort  $n$  nodes by latitude  
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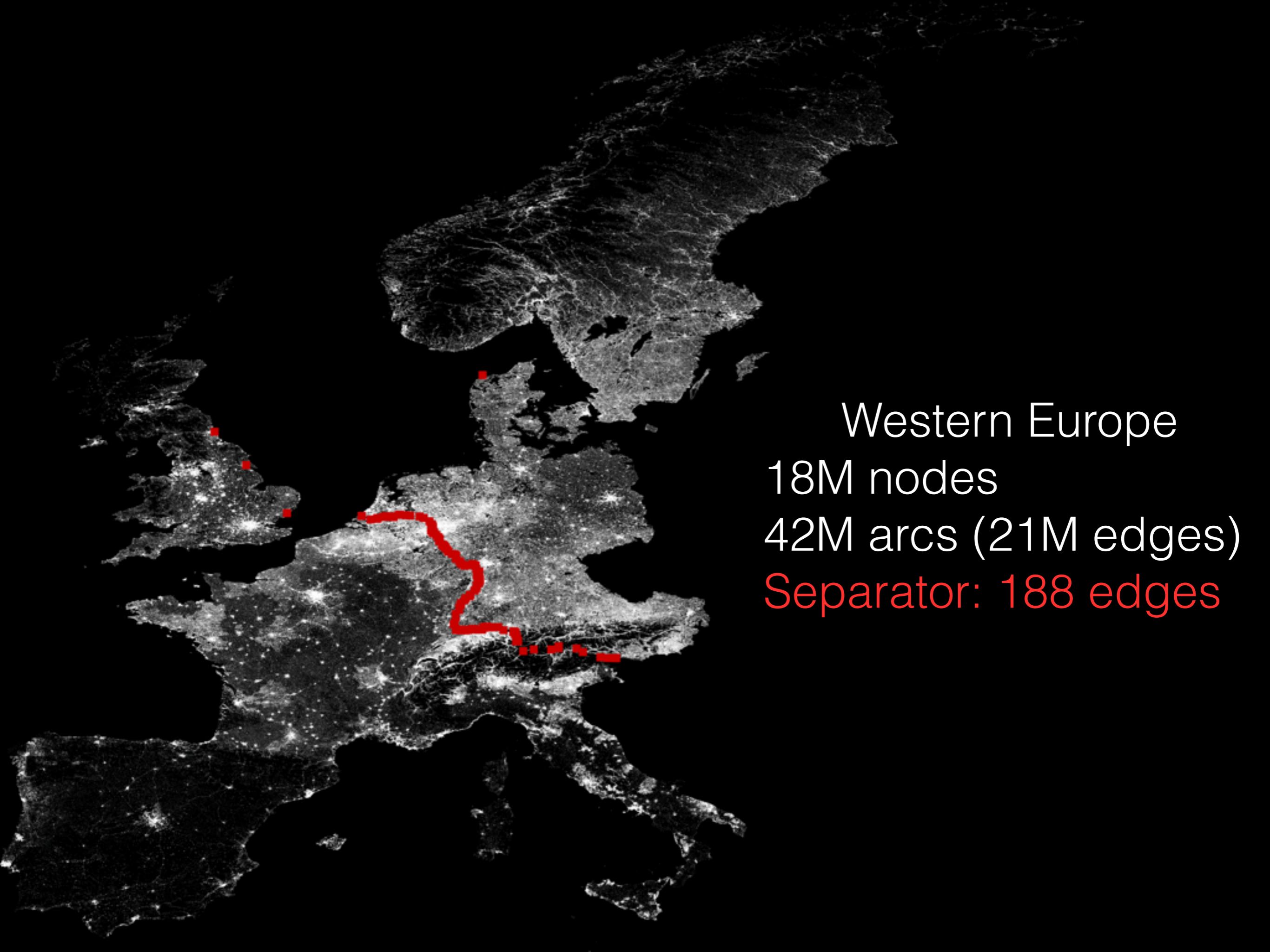
balance (say  $b=1/4$ )

$bn$  sources

$bn$  sinks

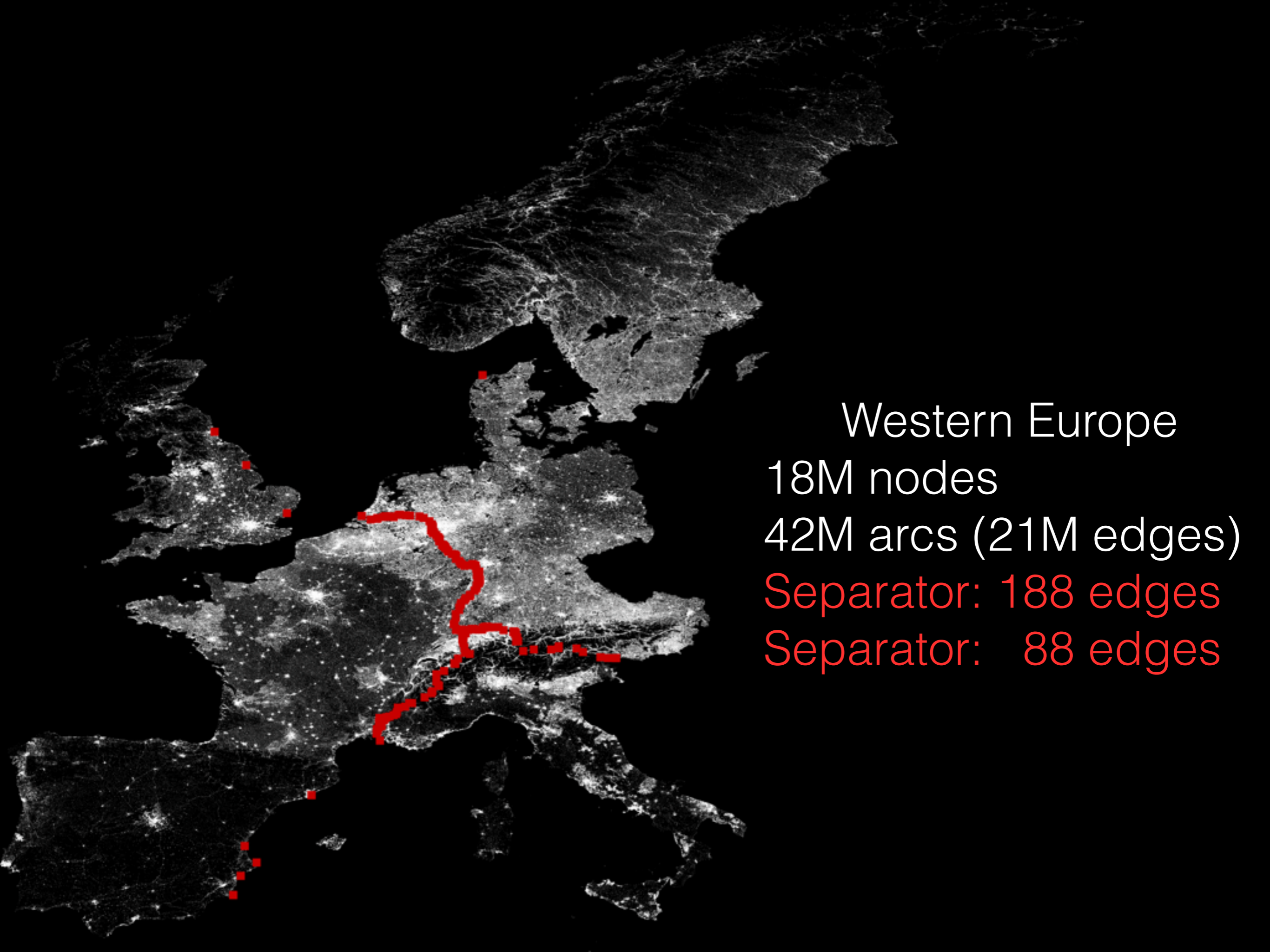
max flow / min cut

recurse



Western Europe  
18M nodes  
42M arcs (21M edges)  
Separator: 188 edges





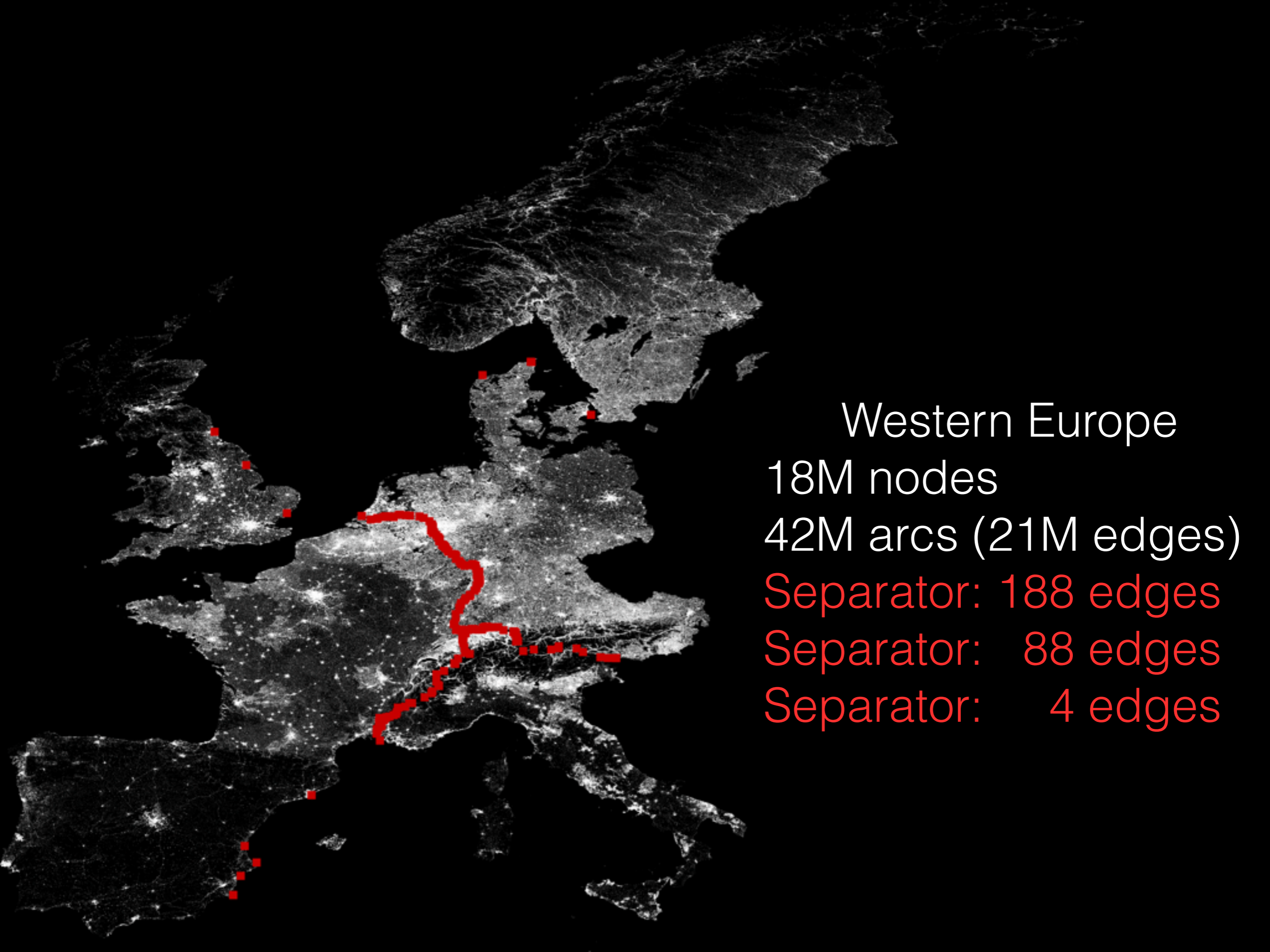
## Western Europe

18M nodes

42M arcs (21M edges)

Separator: 188 edges

Separator: 88 edges



## Western Europe

18M nodes

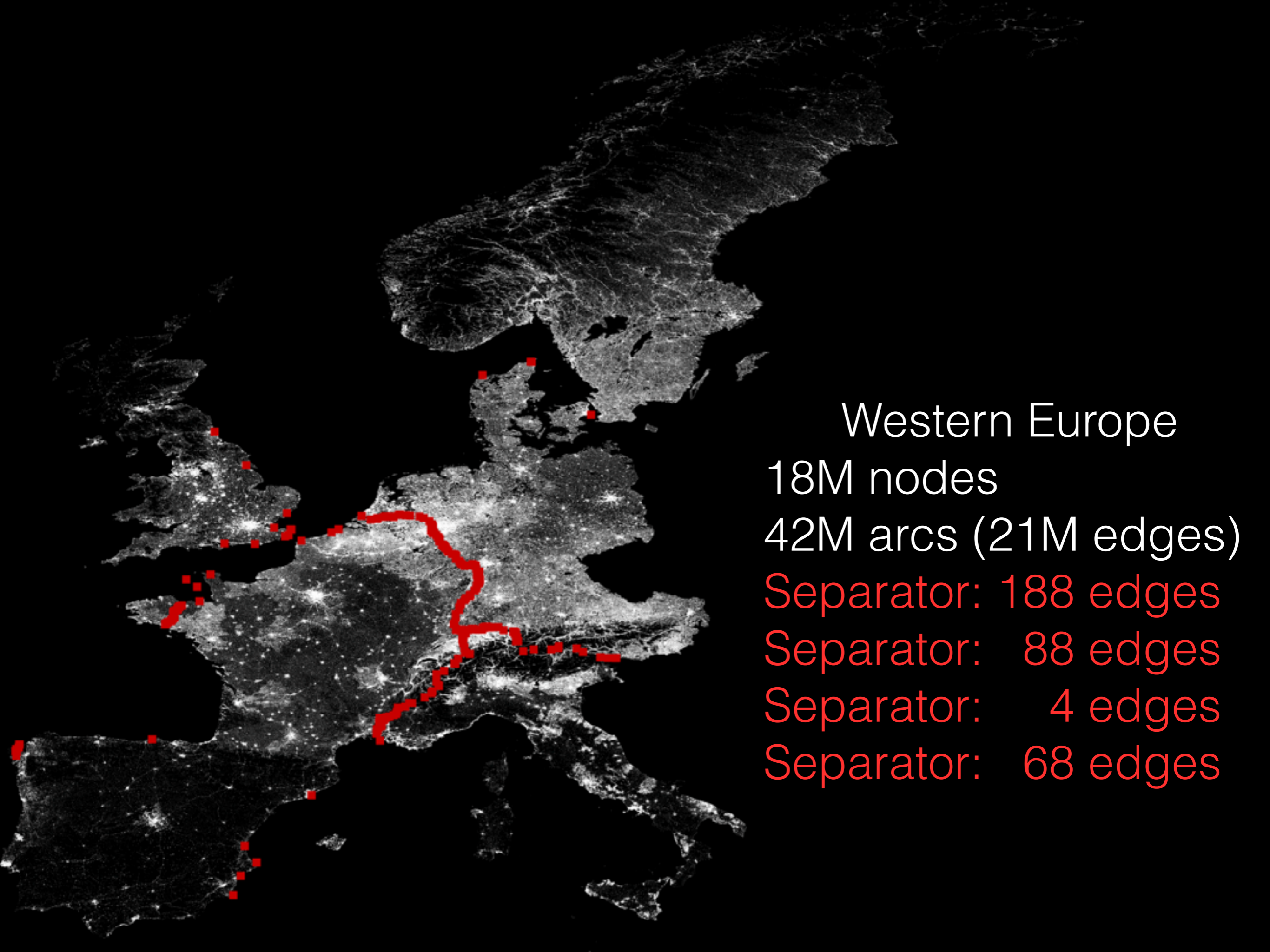
42M arcs (21M edges)

Separator: 188 edges

Separator: 88 edges

Separator: 4 edges





## Western Europe

18M nodes

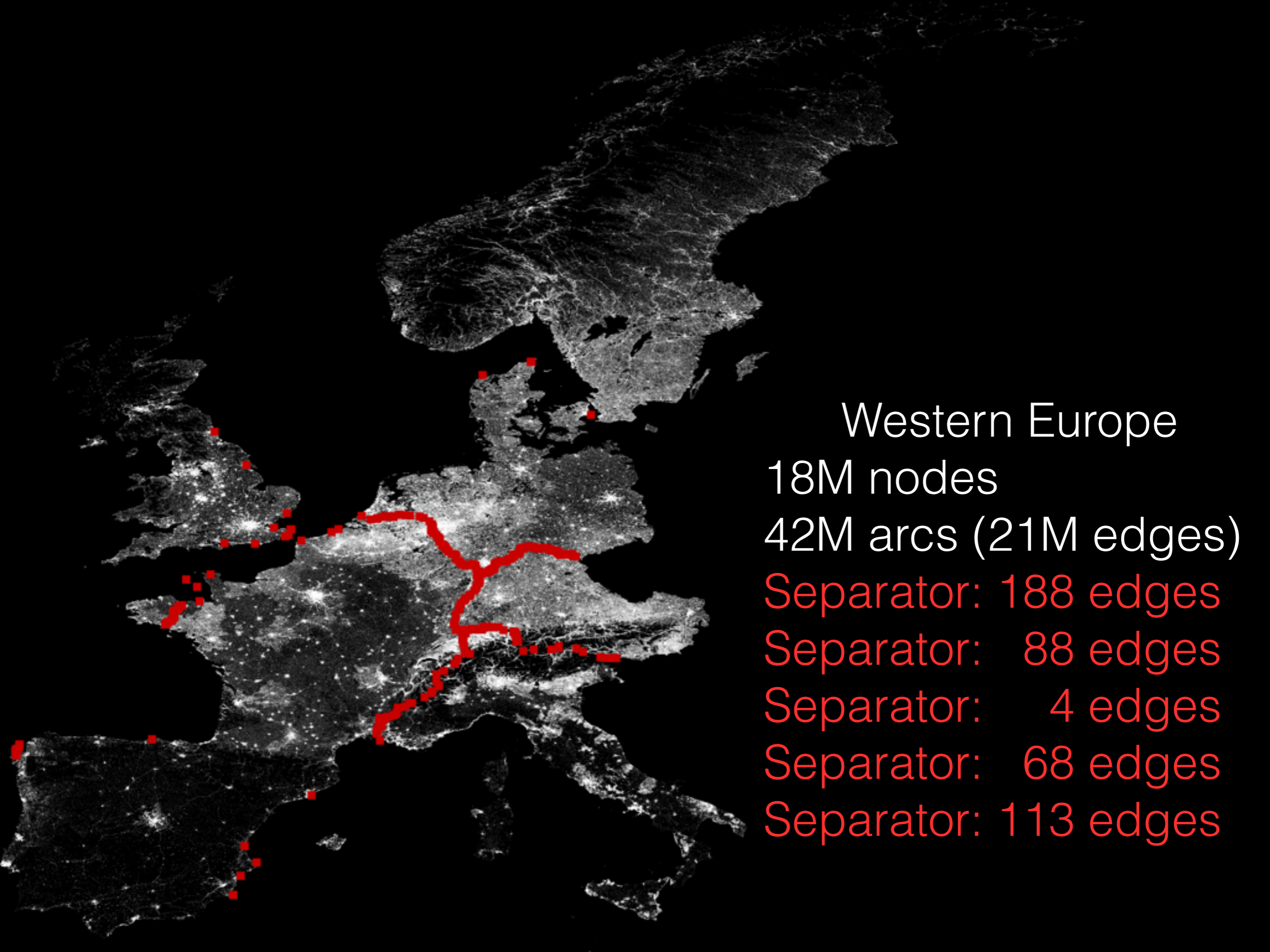
42M arcs (21M edges)

Separator: 188 edges

Separator: 88 edges

Separator: 4 edges

Separator: 68 edges



## Western Europe

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42M arcs (21M edges)

Separator: 188 edges

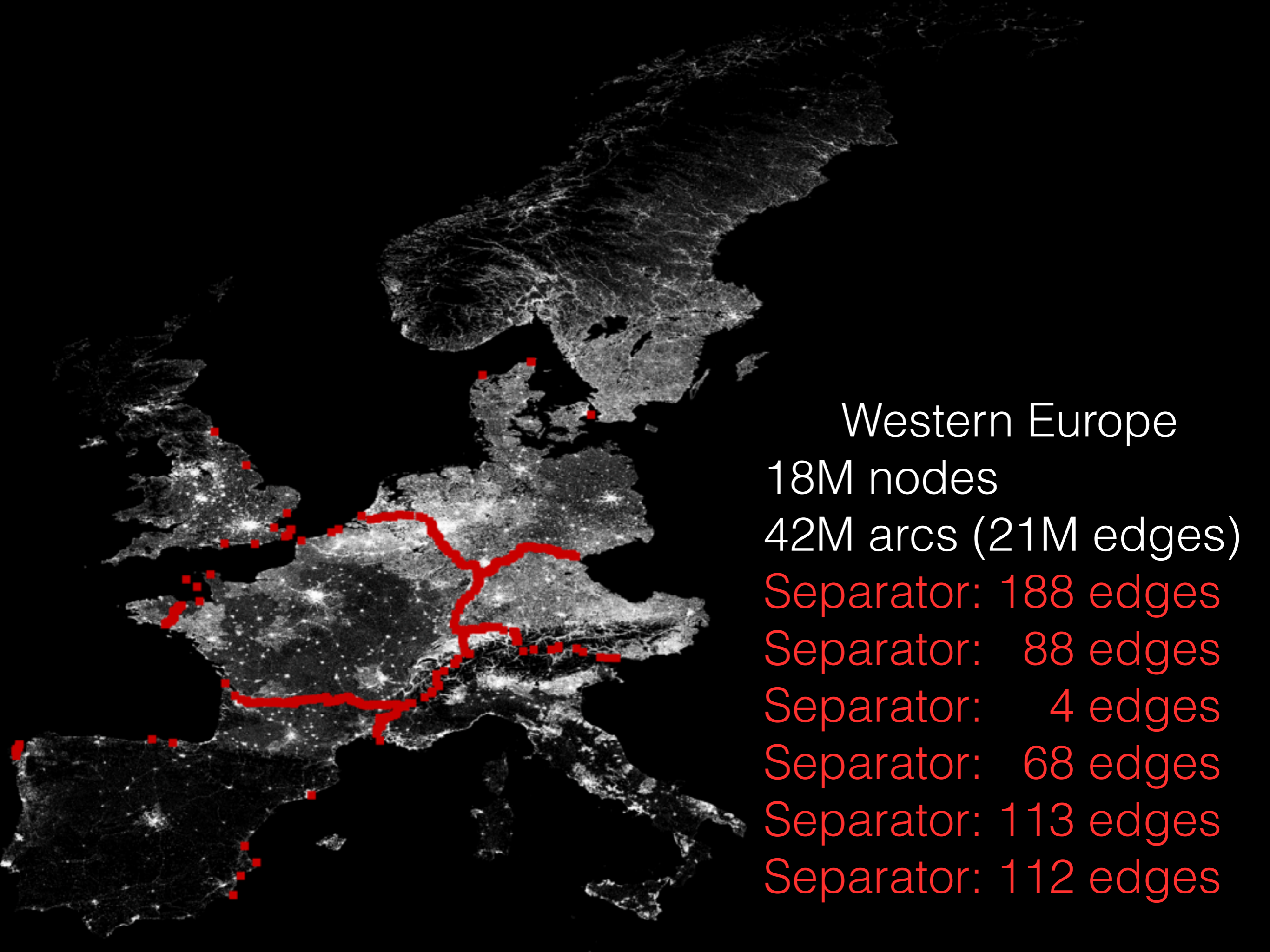
Separator: 88 edges

Separator: 4 edges

Separator: 68 edges

Separator: 113 edges





## Western Europe

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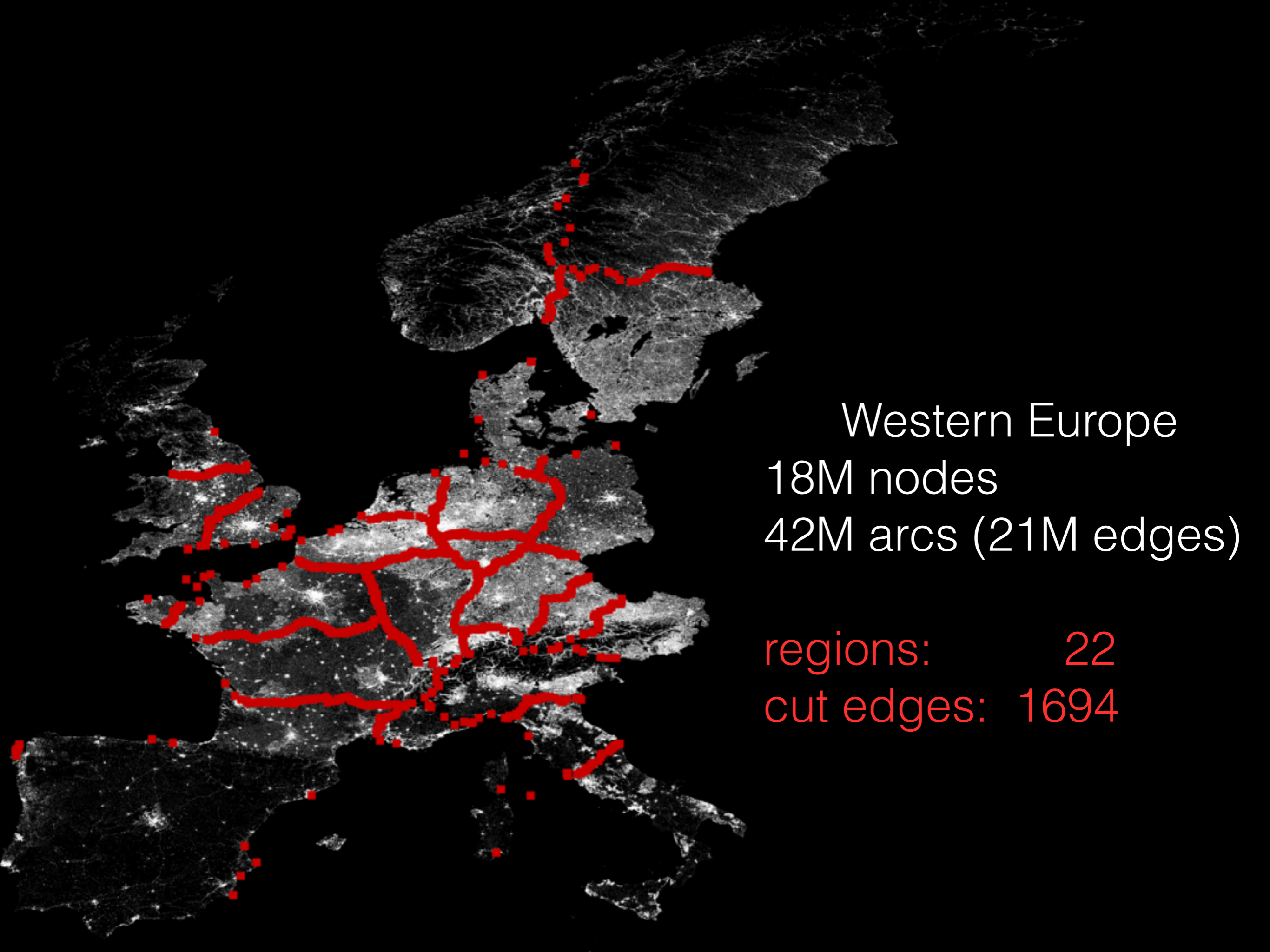
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Western Europe  
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42M arcs (21M edges)

regions: 22  
cut edges: 1694



	PUNCH [DGRW11]		Inertial Flow	
region size $r$	regions	boundary	regions	boundary
1,024	20,129	168,767	27,129	208,280
4,096	5,000	69,304	6,808	84,291
16,384	1,248	28,448	1,708	34,839
65,536	314	11,403	431	14,054
262,144	81	4,194	106	5,275
1,048,576	22	1,464	28	2,036
4,194,304	6	371	7	573

Europe, 18M nodes, 21M edges

	PUNCH [DGRW11]		Inertial Flow	
regions	region size $r$	boundary	region size $r$	boundary
20,129	1,024	168,767	1,378	171,064
5,000	4,096	69,304	5,536	69,016
1,248	16,384	28,448	22,367	28,236
314	65,536	11,403	88,856	11,317
81	262,144	4,194	349,449	4,246
22	1,048,576	1,464	1,299,633	1,694
6	4,194,304	371	4,861,623	461

Europe, 18M nodes, 21M edges

	PUNCH [DGRW11]		Inertial Flow	
regions	region size $r$	boundary	region size $r$	boundary
26,725	1,024	222,636	1,389	223,531
6,643	4,096	87,762	5,570	87,193
1,661	16,384	34,345	22,310	34,138
418	65,536	12,767	87,960	12,971
109	262,144	4,556	336,843	4,557
27	1,048,576	1,504	1,407,053	1,413
7	4,194,304	383	4,338,122	388

USA, 24M nodes, 29M edges



# PUNCH

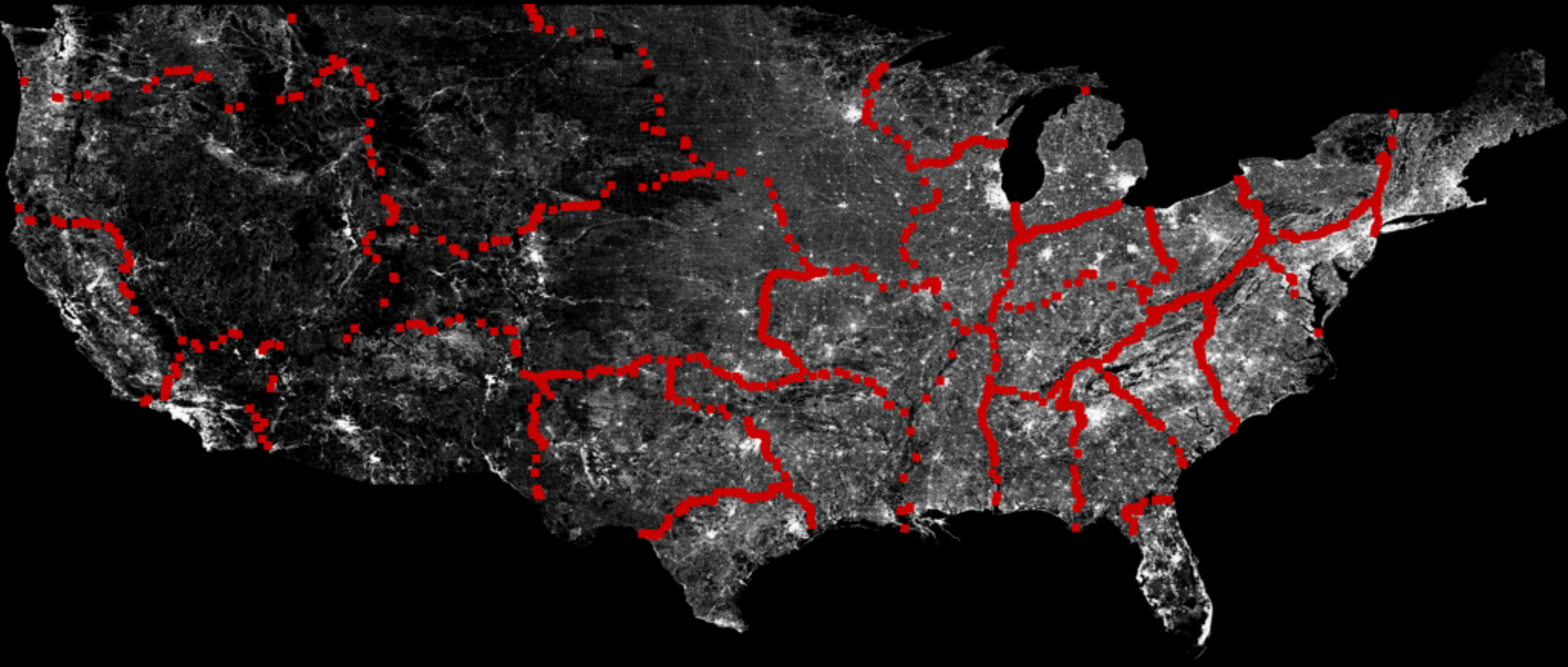
[Delling, Goldberg, Razenshteyn, Werneck 2011]



27 regions, 1,404 cut edges



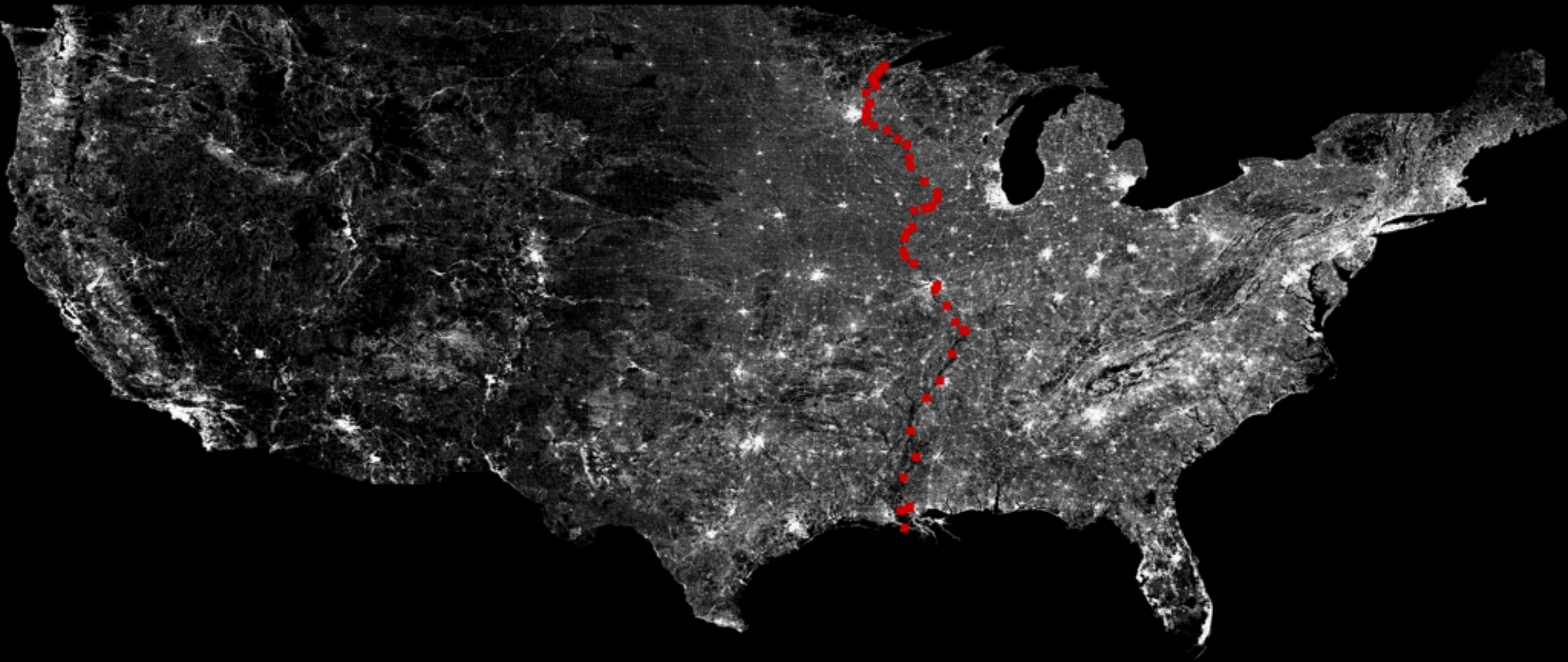
# Inertial Flow



27 regions, 1,413 cut edges



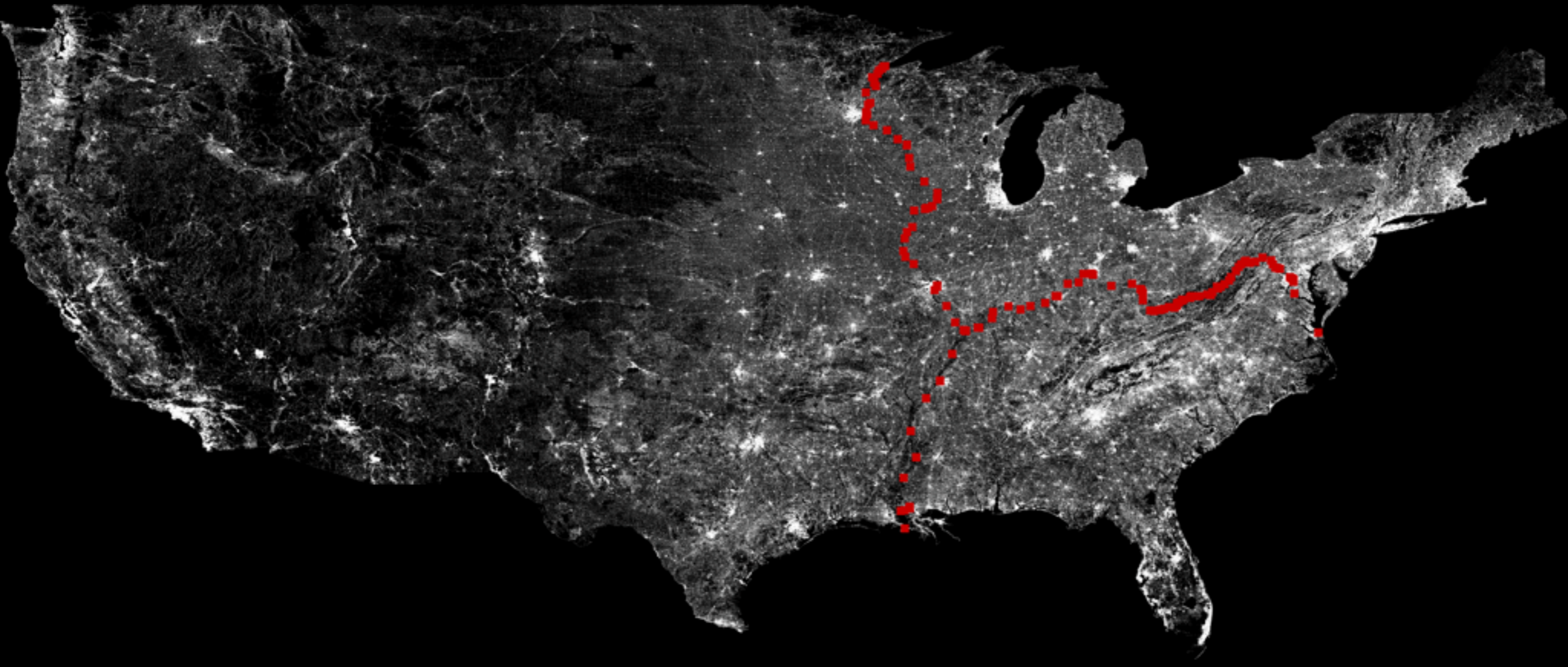
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27 regions, 1,413 cut edges



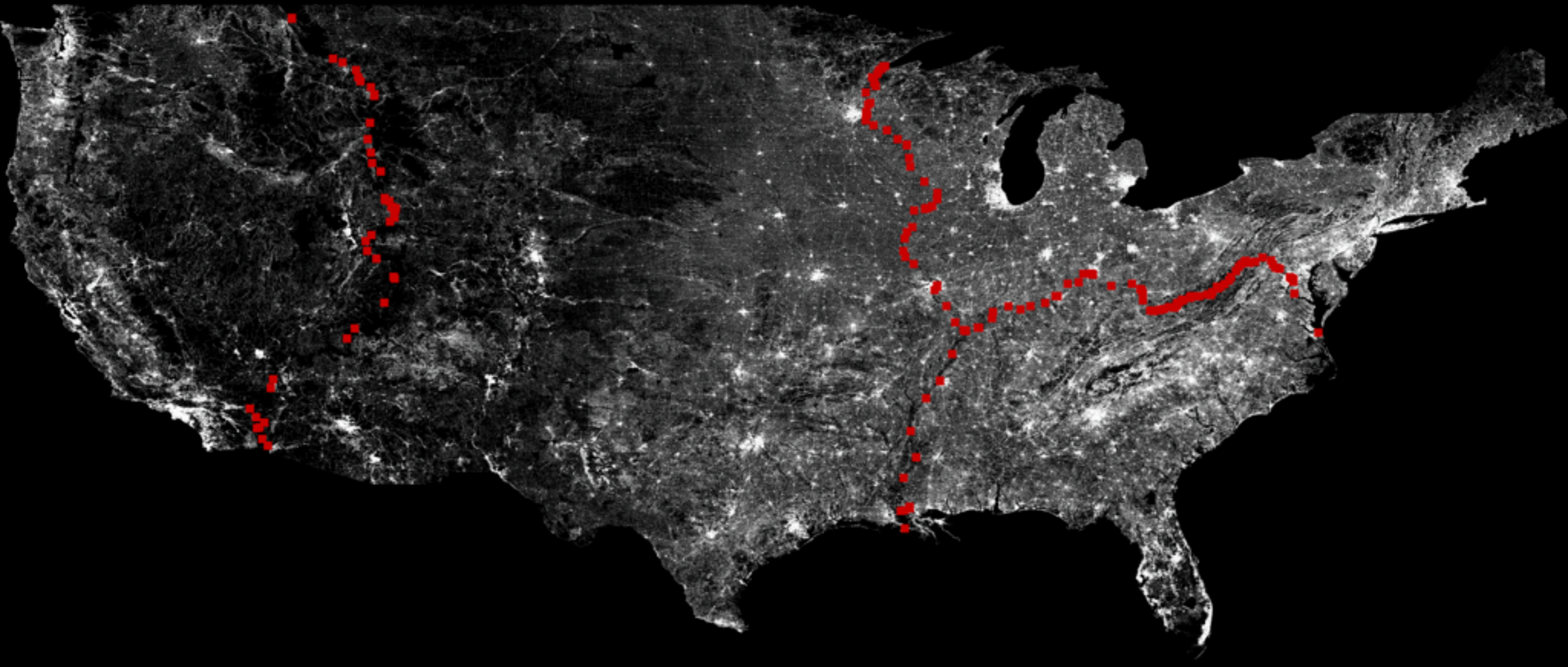
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27 regions, 1,413 cut edges



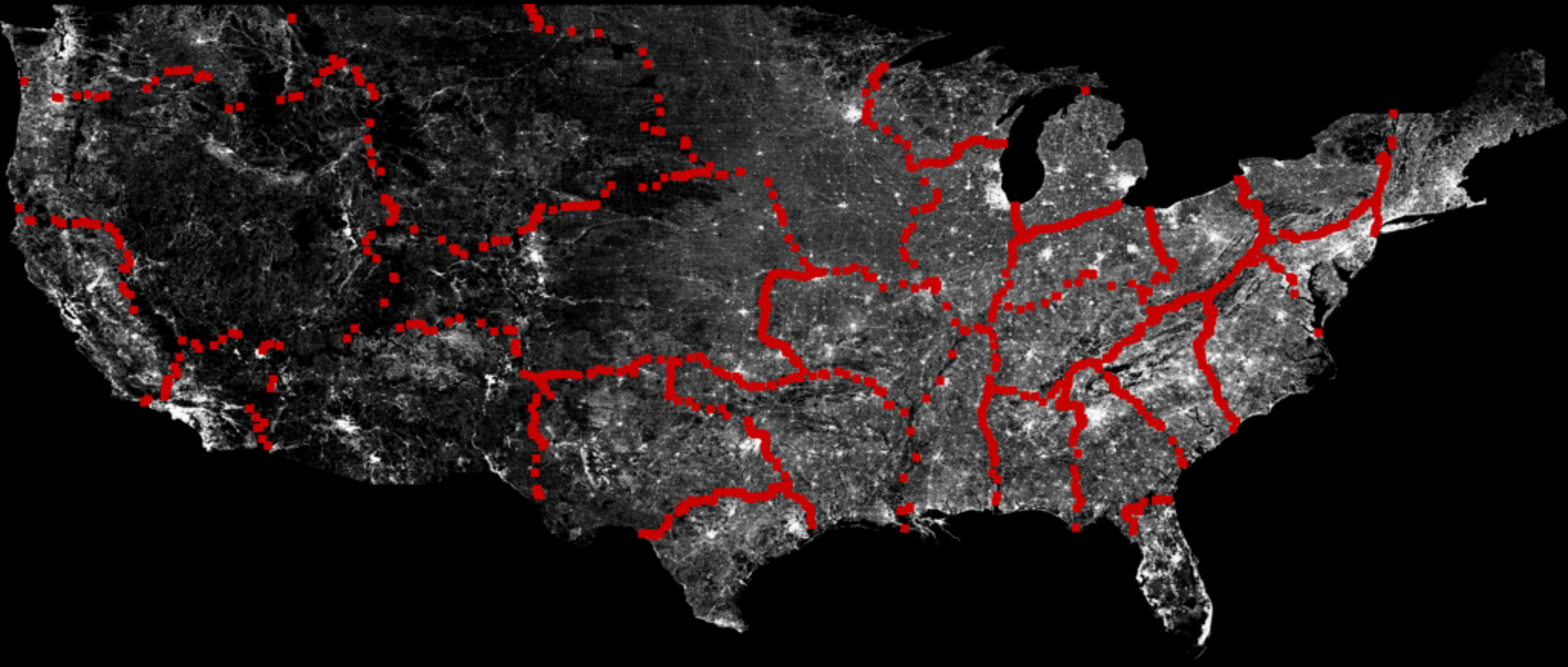
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27 regions, 1,413 cut edges



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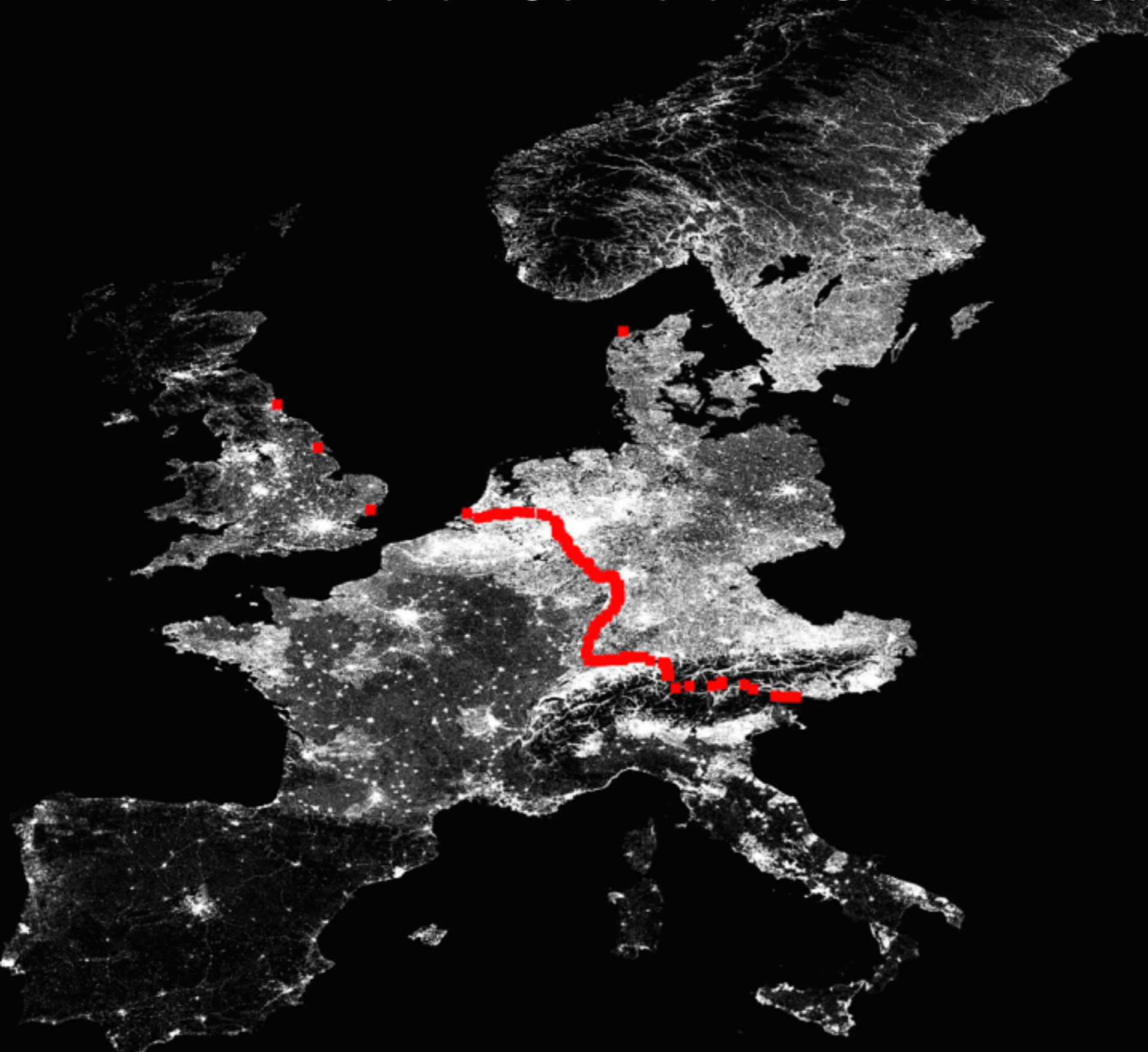


27 regions, 1,413 cut edges



# On Balanced Separators in Road Networks

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Thank you

Merci

Questions?